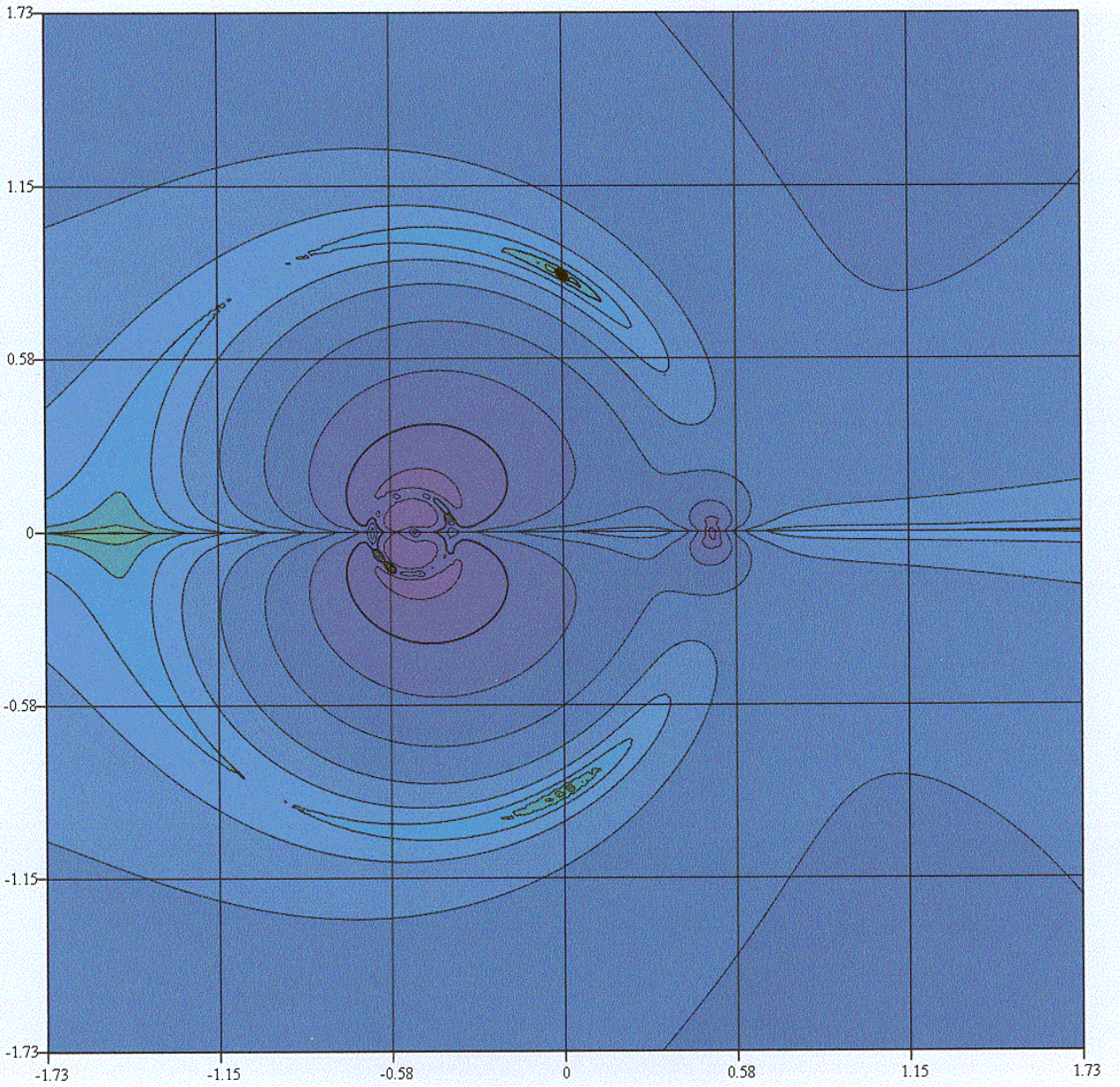


NEWTON'S GREAT… OVERSIGHT

Michael Hugh Knowles

[This page intentionally left blank.]

mhk@mhknowles.net

NEWTON'S GREAT… OVERSIGHT

Galileo's Falling Bodies, Newton's Theory of Gravity, and Lagrange's Trojan Points and Trojan Asteroids with Their Tadpole and Horseshoe Orbits

Michael Hugh Knowles

November 7, 2011

Cover image: Contour plot of a simple function of the falling rate difference of lighter and heavier bodies (a static, 2-dimensional case). One can see regions relating to the stabilities and instabilities associated with Lagrangian points L1-5. The plot suggests that there might be more to the Lagrangian point story than Lagrange discovered, even more than we currently think. (Plotted with MathCAD 2000.)

The basic material for this book—"Newton's Great… Oversight" (NGO)—was first posted on the Internet in 1995. It has been subsequently revised a number of times, with the contour plots finally added in 2000. Since then the technical content has had no major revisions and only a few minor revisions updating Trojan asteroid info. But the overall presentation has been revamped several times; and the philosophical analyses and other maunderings have been extended and reworked a number of times.

This (pre-publication, informal peer review version) ebook is
intended for and dedicated to "stoutly gifted children of any age"
who will see it as a **challenging opportunity for the future evolution of science**:

"Science, right or wrong; when right, to be kept right; *when wrong, to be put right*."

FAIR WARNING: this ebook contains much that some people will find provocative, controversial, polemical, even obstreperous, one way and/or another, e.g.:
"… Science is still evolving in the direction of 'Religious' intolerance…
… Science is greatly in need of 'to be put right'…"

FEEDBACK: I am hoping *very much* for high-quality informal peer review FEEDBACK.
Please, send your FEEDBACK, ERROR REPORTS, COMMENTS and SUGGESTIONS to Michael at:

mhk@mhknowles.net

I am *especially* hoping for GIFTED FEEDBACK (from gifted kids and their gifted teachers).

Michael Hugh Knowles
Rua Miradouro, 45
Sion
Belo Horizonte, MG
Brasil 30310-640
mhk@mhknowles.net

Science, right or wrong;
when right, to be kept right;
when wrong, to be put right.

“I do not say that John or Jonathan will realize all this;
but such is the character of that morrow which mere
lapse of time can never make to dawn.
The light which puts out our eyes is darkness to us.
Only that day dawns to which we are awake.
There is more day to dawn.
The sun is but a morning star.”

Henry David Thoreau (1817-1862)
the closing words of ***Walden; or, Life in the Woods***

[This page intentionally left blank.]

TABLE OF CONTENTS

CHAPTER CONTENTS PREVIEW OF THIS BOOK

CHAPTER 1, Introduction (p. 23):

After VARIOUS WARNINGS, we will expand on the prefatory material already presented, with repetition of the most important essentials.

CHAPTER 2, History**, (p. 29):**

Extremely short bios of the major players and their roles follow: **Aristotle** (**p. 29**), **Galileo** (**p. 29**), **Newton** (**p. 31**), **Lagrange and His "Trojan Planets"** (**p. 32**), and, to round things out somewhat, a very short digressive bit about **Poincaré's Further Work on the 3-Body Problem, and Chaos Theory** (**p. 34**), and, almost of course, a short bit about **Einstein and His "Relativity"** (**p. 34**).

CHAPTER 3, **Newton's Gravity and Lagrange's Trojan Points, (p. 37):**

We will look at **Newton's Theory of Gravity** (**p. 31**), **The "Gravity" of Falling Apples…** (**p. 37**), **"Infinitesimals" and Levels of Approximation** (**p. 41**), **Some Basic Equations and Some Simple Equations** (**p. 44**; Newton's, concerning the difference in falling rates in the case when e.g. a lighter and heavier apple are *separately* released). **Section 3.5 Equations for a *Simple* 3-Body Problem** (**p. 47**) concerns the simultaneous release case. It contains the fundamental set of equations, which use only algebra and trigonometry. The fundamental result is that the 2 bodies will simultaneously fall to Earth at the same rate *if* they form an equilateral triangle with the Earth. We will also take **A Quick Look at the Separate Release Case… and Einstein's "Relativity"** (**p. 52**; we show why the result of different falling rates merely *seems* to contradict Einstein's general relativity).

We extend this fundamental result to a more general result, as did Lagrange. It is shown that the equilateral triangle formed by the lighter and heavier bodies and the Earth (actually, any 3 "point masses") can be set revolving around the common center of mass in a stable orbit (abstracting out "perturbations" and their effects), even with the equilateral triangle oscillating in a cycle of expanding and shrinking. (After this there is hardly any more mathematics per se.)

We give simple arguments for the "stability" of the triangular formation when the masses are perturbed, but acknowledge that this stability needs more in depth study to try to find the kind of simple demonstration of it that even Newton's lesser contemporaries could have developed. It is likely to be difficult to simplify it to that extent because it will become obvious that the global stability at either equilateral triangle point, L4 or L5, depends not just on the other point but on L3, as well. Looking at **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140** may make this obvious to some. The punctuatedly evolving concept of "stability" is looked at from a historical-philosophical perspective.

CHAPTER 4, **TROJAN POINTS AND THEIR TADPOLE AND HORSESHOE ORBITS (p. 67):**

We shift to the world of astronomy. We talk about Trojan asteroids and their fascinating orbits, which are actually rather more complex than the "tadpole" and "horseshoe" orbits that popular astronomy talks about. Suggestions are made relating to possible spacecraft exploration of the Trojan points of the Moon and the Earth (because they are so near) and our other planets.

CHAPTER 5, TROJAN ASTRONOMY IN THE 20TH AND EARLY 21ST CENTURIES (p. 71):

We will take a brief look at the history of Trojan point astronomy. First we will quickly review some of astronomy's discoveries concerning Trojan asteroids, a field that has become very popular in recent years—especially since 1999, with the discovery of the Earth companion 3753 Cruithne (see **Section 4.2, Trojan Asteroids**

in Horseshoe Orbits, p. 67, and **Section 5.2, Trojan Point Astronomy in the 20th and Early 21st Century, p. 71**). Then we will offer suggestions about how the (overly, to be sure) simple approach to Trojan points presented here might help inspire and facilitate yet further study and understanding of Trojan dynamics.

CHAPTER 6, NEWTON'S—AND SCIENCE'S—Great "… Oversights" (p. 79):

We will start taking science to task for *some* of its important *scientific* failures, not the failures usually pointed out and analyzed by critics who are non-scientists. This is where we enter "Science Wars" territory, almost a philosophical Twilight Zone. There is an attempt at analyzing the historical and psychological sources of "Newton's Great… Oversight". There is also quite a bit of potentially polemical and even obstreperous philosophizing about the whole scientific "… oversight" situation, especially when those issues relate to the so-called "Science Wars".

CHAPTER 7, Toward New Science and New Philosophy of Science (p. 94):

This chapter contains the vast majority of the philosophical maunderings and diatribes of this book. There are a lot of relatively cute quotables, as well as ideas you can try to philosophically one-up your friends and colleagues with, such as the idea that "truth" is not a property of a map (or a model, or a statement, or an abstraction, or what-have-you), but a property of the territory of reality which the map is attempting to… uhh, map or model or state or what-have-you. It looks to a future where we become wise with regard to the inherent limitations of science, mathematics, logic, and other means we have so far evolved to try to capture truth like a cat captures a bird or butterfly (or more likely, a cockroach). It looks to a future where science isn't pervaded with "… oversights" like "lighter and heavier bodies always fall at precisely the same rate".

APPENDIX (p. 129)

FIGURES referred to in the text:

Figure 1: The 3 Bodies/Masses, and the Angles and Distances Among Them (p. 136)

Figure 2: The Difference in Falling Rates as a Function of Angular Separation (p. 137)

Figure 3: The 3 Masses and 2 of Their Centers of Mass (p. 138)

Figure 4: The Lagrangian Points L1-L5 (p. 139)

Figure 5: "Tadpole" and "Horseshoe" Orbits (p. 140)

Figure 6a: Are Stable Trojan Star Systems Possible?! Part a (p. 142)

Figure 6b: Are Stable Trojan Star Systems Possible?! Part b (p. 144)

ABOUT THE AUTHOR (p. 146)

ABOUT THE AUTHOR–FOR "STOUTLY GIFTED CHILDREN OF ANY AGE" (p. 147)

[Back Cover] (p. 151)

Use of Bullets to Mark Text

An explanation of the use of "bullets" for marking text:

- ➢ Used to mark emphasized (more important) main points.
- ✓ Used to mark inline what would otherwise be an (important) endnote or footnote.
- ❖ Used to mark emphasized *digressive* points, i.e. important supplementary material.

PRELUDE

Newton's great insight into gravity—his Great "Aha!"— was that,
not only does gravity make the apple fall toward the Earth,
but gravity makes the Earth fall toward the apple!
If we have 2 masses, 1 lighter and 1 heavier, $m_L < m_H$, and Galileo releases them
at separate times from the top of the Tower of Pisa, the Newton-theoretical falling rates
are the *combined* accelerations of each mass and the Earth toward each other, i.e.

the **lighter mass falling rate** = $G\frac{m_E}{r^2} + G\frac{m_L}{r^2}$ < $G\frac{m_E}{r^2} + G\frac{m_H}{r^2}$ = the **heavier mass falling rate**

and we can see that the lighter and heavier bodies have Newton-theoretically *different* falling rates.

But there's more…

Simple Newtonian theory of gravity
+
simple algebra
+
simple trigonometry
+
"*scientific heresy*"
(in questioning Galileo's *scientific* finding
that lighter and heavier bodies fall at the *same* rate,
that for some strange reason we all still believe is scientifically correct)
=
a *simple* proof that Newton's theory of gravity *predicts*
that, when released *simultaneously* from the top of the Tower of Pisa,
lighter and heavier bodies *must* fall at *different* rates (*usually*)
+
a *simple* approach to the alluring astronomy of Lagrange's Trojan asteroids
and their *fascinating* "tadpole" and "horseshoe" orbits
around their corresponding Lagrangian/Trojan points, L4 and L5,
without using Lagrange's *exceptionally difficult* perturbation theory,
without using Lagrange's *exceptionally difficult* calculus of variations,
without using *extremely difficult* partial differential equations, and
without using *even simple calculus!*

And We Have a Novel Twist in the Modern Science Wars:
***Isaac Newton*—and Now *Modern Science*—Failing on *Scientific* Grounds…**

IMPATIENT?!

WHERE CAN YOU FIND THE EQUATIONS?!

Section 3.4 Some Basic Equations and Some Simple Equations starts on **p. 44**.
Section 3.5 Equations for a *Simple* 3-Body Problem starts on **p. 47**.
Section 3.9 Equations for Homographically Maintaining an Equilateral Triangle Without Expansion and Contraction starts on **p. 54**.

You, too, may find yourselves asking:

"They missed *that?!* Then what *else* has Science missed?!"

PROLOGUE

Isaac Newton (1642-1727), often counted as the greatest scientist who has ever lived, made an ***incredible* "… oversight"** in his theory of gravity:

- **Newton overlooked that his *very own* theory of gravity predicts that lighter and heavier bodies will fall at *different* rates**—with a scientifically fascinating exception to be detailed in this book.
- ***Before* you have a stylishly fashionable intraparenchymal hemorrhage or your classic myocardial infarction, remember…**

  Newton's "Great *Insight*" into gravity:

  Not only does gravity make the Apple fall toward the Earth, but gravity *also* makes the Earth fall toward the Apple. The Earth will fall faster toward a heavier Apple than toward a lighter one. So when two Apples are released by Galileo *one at a time* from the top of the Tower of Pisa, and the rate of fall instantaneously measured at the base of the Tower by Newton himself, the heavier Apple will be measured as falling slightly faster, just as Newton's theory predicts.

  Things get even more interesting when both are released simultaneously! This is the scientifically fascinating exception mentioned earlier, to be detailed later in this book. The physics and math are so simple that even Newton's lesser contemporaries could have/should have discovered this.
- **But as incredible as that may seem, it is even *more* incredible that even today, in/on the year-month-day 2011-11-07 CE, more than 300 years after Newton's *Principia Mathematica* and his universal law of gravity, *we still believe scientifically* and *we still teach in our schools—as science*—that, per *Galileo's* findings, lighter and heavier bodies fall at precisely the same rate.**

  How can this be?!

  It is this double mystery that in 1995 inspired the conception of this book.
- **This is the modern "Science Wars" but with a new twist:** not science failing "merely" on "unscientific" grounds, but **science failing *scientifically*,** in its *own* territory, on its *own* terms, a result everyone who loves or is affected by science in this modern world of ours should be interested in.

Newton made (at least) three great oversights, only two of which have been publicly acknowledged to date. Everyone who has studied calculus knows that Newton overlooked Leibniz's mathematically inspiring and now standard $\mathbf{d}f(x)/\mathbf{d}x$ notation for his own famous variant of the calculus. And, as many scientists and "popular scientists" know, Newton overlooked the wave nature of light; but far worse, he used his by then preeminent reputation and position to scientifically crush his wave-hypothesis "enemies" (for so he deemed them), helping to keep the physics of light in the "Dark Ages" for almost 200 years, until James Clerk Maxwell (1831-1879) developed his theory of electromagnetic waves. (If you are sharp, you may already have thought of Thomas Young and his interference experiments in the very early 1800s, and of Fresnel.) The third, as mentioned in the first paragraph, above, is the focus—or rather the focusing lens—of this book.

I first began to fully appreciate the scientific and educational importance of what I term "Newton's Great… Oversight" (NGO), for so I deem it, in the middle 1990s. Poor health had forced me to retire from a Silicon Valley software development career, and I eventually returned to studying my childhood loves of science, mathematics and philosophy. My love of science is richly of the kind:

- Science, right or wrong;
  when right, to be kept right;
  when wrong, to be put right. [my special emphasis]

I have a "talent" for finding things that others overlook, not always positively appreciated, as one might guess, but one that stood me in good stead for decades as a software architect/engineer. I decided to aim it at the foundations of science and mathematics, to see if I could find anything that others had overlooked, and… I did.

When I first published my findings on this oversight of Newton's on the Internet in 1995, I received two responses that affected me especially greatly:

First, I received… *anger*.

Shortly after first publishing my results on the Internet, I got a flame e-mail from (a traditionally nameless) someone who said he was the head of a major scientific laboratory. He asked me—he was obviously *very* angry—(parasummarizing in quotes) "how *dare* you find fault with established physics, especially physics that has been so *completely* accepted since Newton?!" I diffidently insisted that he review my equations, which—*to my utter amazement*—he did. He then very grudgingly admitted that the equations were correct, and that lighter and heavier bodies fall at different rates (with the special exception to be detailed later). BUT, he also insisted, "the difference is not really very great and therefore is not really scientifically important." I responded that the advance in the perihelion of the orbit of Mercury is not really very great, either, and indeed is often described as "infinitesimal", but that it is considered VERY scientifically important (for Einstein's relativity). Our e-mail correspondence ceased abruptly.

The second response was from a high school teacher who said that one of her students had found my web site and asked about it in class. She wanted to know "what to say" when her students asked her if lighter and heavier bodies actually fall at the same rate or not. At first I tried to explain how it worked, that if Galileo had dropped two "apples", let's say of 1 and 2 kg, from the Tower of Pisa, because of the gravitational asymmetries they would fall at *slightly different* rates, whether released simultaneously or separately. After several such "explanatory" e-mails she finally pleaded with me, "but is it OK if I tell my students that lighter and heavier bodies fall at the SAME rate?" I gave up and reassured her that they since they fall at *approximately* the same rate (the difference is roughly in the range of 5 parts in 10^{19} for the simultaneous release case and 2 parts in 10^{25} for the separate release case), she could say that. But I felt as though I had betrayed her, betrayed our educational system, and even betrayed science itself.

I received other e-mails of this sort about my Newton—and other—web pages: angry e-mails, doubting e-mails and such, but these two stand out in my memory. I also received some very gratifying positive feedback from people all around the world. Together they all represent why I have come to feel such a sense of need to get this book written and published, a passionate need to get "Newton's Great… Oversight" publicly acknowledged, not just within the scientific community, but especially within our educational system.

It is my hope that NGO—upon the community acceptance of "Newton's Great… Oversight"—will eventually form "paradigms", more or less in the sense of Thomas Kuhn (1922-1996; author of *The Structure of Scientific Revolutions*), but in the arena of "Science… when right, to be kept right; when wrong, to be put right." These will not be paradigms for *maintaining* "normal science" in power, but rather for continuing to hold a "Sword of Damocles" over the head of "normal science" ("when right, to be kept right"), and letting it do its thing—fall—as appropriate ("when wrong, to be put right"). This will also fulfill the ideal of theoretical "falsification" that Karl Popper (1902-1994) made a staple of his version of "logical positivism" and of the role of such "falsifications" (Popper's scientific philosophy and terminology) in "scientific revolutions" (Kuhn's terminology) or, as I suggest "scientific punctuated evolutions" or "punctuated scientific evolutions".

- ✓ I used to say—and I humbly admit that it is terminally cute besides being mildly insightful—that "revolution is evolution as seen by the near-sighted", but some cases of "punctuated evolution" can be difficult to foresee, even by the "far-sighted". And there are probably very useful insights that can be perceived and communicated better by allowing pertinent distinctions to be made from time to time between the terms "punctuated evolution" and "revolution".
- ✓ By the way, "punctuated evolution" is actually my variation on the term of "punctuated equilibrium" coined by Stephen Jay Gould (1941-2002) in a 1972 paper by Niles Eldredge (1943-) and Gould (who credited Eldredge with most of the important ideas presented in that paper). I hold that the term "equilibrium" will eventually strongly fall out of favor in science—except as a purely gedanken concept—when it is realized and accepted that it is a *purely*/*pseudo* gedanken concept, and cannot be otherwise since "equilibria" *cannot* in fact exist in nature *as we theoretically conceive of them*, and we run afoul of the logically dreaded reasoning from false premises when we use these gedanken concepts as if such things exist in reality. See Karl Popper, *The Logic of Scientific Discovery*, for more on his logical positivism. See Stephen Jay Gould, *The Structure of Evolutionary Theory*, for lots more the subject of "punctuated equilibrium" and its role in evolution.

PRE-SCRIPT

Our educational system often receives criticism for failing to produce graduates (at any level) who have the scientific and mathematical sophistication that other countries' graduates have, and all too justifiably. We certainly have the ability to eventually produce graduates who are as sophisticated as should be desired, but as we improve our educational system we should take care not to produce such graduates who have had their scientific imaginations aborted in the process, which latter criticism is rarely leveled against the graduates and the educational systems of those countries that are so often pointed to as superior to our own American system of education. But we fail to produce them because, among other reasons, we have a tradition of never offering education that is aimed any level significantly higher than the "lowest common denominator", as if this will somehow per se bring about our recent ideal of "no child left behind". In fact, we have the unenviable situation that all our children except the lowest common denominators are left behind, and even they don't really get ahead as much as they should.

The reason this is relevant here is that I intend to aim NGO at quite a range of denominators, far from just the "lowest". I intend it to challenge the curiosities and imaginations of even profoundly gifted children, even if it is also accessible to anyone at a high school physics class level or with equivalent popular science background. This way I get to indulge my eccentric sense of humor in the midst of what might normally be material presented with dry didacticism and petrified pedagogy. I do this not merely to entertain myself and those who can keep up with it in ways relevant to the material itself, but because of the educational advantages that come from making the all the material, basic and advanced, more memorable as well as more accessible.

NOTA BENE: if the "going gets too tough" in what follows, especially the next section, feel free to just skip ahead to whatever interests you. I can get very wordy about my pet peeves and such.

PREFACE

As of this writing, we have lived for quite some time—and we can count on living for quite some time in the future—in an age where "basic scientific literacy" rivals in importance what has long been considered "basic literacy/education", the traditional "3 Rs"—"Readin', 'Ritin' and 'Rithmetic". It is no longer enough for all of us to be able to read, write, add, subtract, multiply, divide, and calculate interest and mortgage payments. Our future will of necessity depend ever more importantly on science and its future development, its future *evolution*, *and* on our *shared* understanding of all these—though with no necessity or even desirability of anything approaching compulsory or obligatory unanimity. (Neither ecosystems nor souls can survive, let alone thrive, flourish, and evolve, if forced to fit a Procrustean Bed or Iron Maiden of "unanimity". The number "one" is a *very* strange number…) We all need to acquire—at the very *least*—a basic understanding of science, its history and philosophy, its strengths and (also essential) its weaknesses, especially the inherent ones, and its future potentials for good and for… not so good.

We especially need to know the truth (and be made free by it) that science—like religion before it—will never be perfect, far from it, having no realistic chance to ever become truly "omniscient" or "infallible", even "in matters of scientific faith and morals". Rather the opposite. Our "modern science" is still far from mature. It is still far from "adolescent" or even that all important "as a little child".

It is this author's considered opinion that we should all learn to view "science"—which term supplanted the term "natural philosophy" in the early 1800s, less than 200 years ago—as not yet even having been fully "*conceived*". We are still gathering and sorting through various choices for its "DNA", as it were, the DNA that will form the templates and paradigms—a good term despite the fact that there has been a minor but vocal post-Kuhnian abreaction to it, including from Kuhn himself—for science's future development and evolution. If this seems an exaggeration to some, we can note that it is only in the last century or so (in the early 1900s) that the once purely metaphysical concept of "atomies", that was first proffered (that we know of) by the ancient Greek philosopher Leucippus and his pupil Democritus, finally became accepted as a scientific reality by our "modern science".

The reason for science's immense success—so far—can perhaps best be explained by a quote from (I hope I am remembering correctly) James A. Michener's *Centennial*. In one of the stories, an agronomist is warning the farmers of impending danger from their agricultural practices, and, when they get a good crop anyway, he tells them that this is just the "sod crop" (which term has other uses). He tells them they are taking up the sod of the original prairie of the North American continent, formed over 100s of millions of years, and the soil underneath is so rich that even with bad agricultural practices they will get good crops for a few years. Then the soil will start to give out, and what's left will be washed and blown away (as it later did during the Dust Bowl years of the Great Depression), and disaster will stalk the land… (That "disaster will stalk the land" is *my* somewhat eccentric sense of humor "sprouting"; don't blame Michener.)

Science's immense success so far very much resembles this "sod crop success". We can note that science's formal insistence on "reproducible experiments" has quieted

much in recent years, with many fantastically expensive and long term experiments being considered successful if they "produce even once". Science's Kuhnianly paradigmatic insistence on measurement and quantification being the only legitimate way to obtain "objective knowledge" may—quite soon—find itself yet another "sod crop failure" and "consigned", the victim of yet another "scientific—or meta-scientific—revolution".

Religion—perhaps past the conceptual stage, but still fetal—long ago evolved the attitude of "Religion, right or wrong." Unfortunately, it never (in general) conjointly evolved the essential adjunct of "when right, to be kept right; when wrong, to be put right." Religion too had its "sod crop" era. E.g. it was the priests who predicted the flooding of the Nile and advised the farmers how and when to be prepared for planting and harvesting. Then Religion found itself failing due to "bad land management practices"—or was it due to "bad fishing practices". In fact, this long term failure has been a large part of the impetus for the evolution of not only philosophy and natural philosophy, but of our now "modern science". It is worth another bit of short pre-historical digression here because our "modern science" seems to be following in religion's footsteps to a frightening degree.

It is cute—and may not even be *too* over-simplified—to posit the position that long ago (the "theses" of) religion(s) wound up "out of milk"; i.e. they "ran out of the milk of revelation" because they had "lost faith". The original sacred scriptural meaning of "faith" was *never* "beliefs held despite ignorance", but was rather a "connection", a "communications channel", which makes remarkable sense when we remember that "religion" comes from the Latin root "religare", meaning "to reconnect" or "to rebind". Of the many things that came of this, one of the most important was that this failure of ongoing "revelation" (the lack of essential communications from people's spiritual home base) left a big empty niche, which stimulated the evolutionary dynamics to fill said niche with (at least) one of religion's punctuatedly evolving "antitheses", i.e. "philosophy", which can still be said to be evolving… punctuatedly, or "revolving" in a Kuhnian sense. Philosophy itself soon started evolving a *branch* off into "natural philosophy", the philosophy of nature, i.e. of the "natural world" as opposed to the "super-natural world" of "religion", the ages long attempt to reestablish the "faith", the connection of the original spiritual communications channel (the "lost chord/cord"). "Natural philosophy" (*and* we can say its "paradigms" in the sense of *early* Kuhn) then seemed to disappear, somewhat like Neanderthalensis, with its place being taken by something proclaimed and/or proclaiming to be "science" (along with *its* newly punctuatedly evolved/evolving paradigms) in the early-to-middle 1800s.

We can oversimplify everything by noting that religion evolved to deal with invisible essences—we can borrow Kant's term, "noumena"—while science evolved to deal with visible "phenomena". In our current scheme of things religion has always tended to have little respect for "bodies", rather more visible, and tended to accent "souls", rather less visible. Modern Science unwisely denies the existence of invisible souls and other such entities, even despite the fact that science has started dealing almost exclusively with invisible entities ("forces", "quarks", etc). Religion holds that the life we know more-or-less in common is the purely the product of acts of "creation" by invisible (to us) intelligent spiritual entities that religion ostensibly studies, and claims to act as the only legitimate intermediary for. In science's view, the life we know more-or-less in common is purely the product, the "emergent behavior", of the mechanical—more modernly quantum mechanical and automatological—activities of the nature that science ostensibly studies (noticeably less philosophically than in Newton's day), and claims to act as the only legitimate intermediary for.

Religion needs to become more cognizant of the spiritually significant reality of visible phenomena, and science needs to become more cognizant of the scientifically significant reality of invisible noumena, living and intelligent spiritual essences that are undetectable in our current state of the "emergent behaviors" that make up our current scientific competence.

But no obvious syntheses have yet appeared on the horizon from these contretemps of theses and antitheses, unless like marriage among humans, a "running battle of marital bickering" between religion and science can be considered "some such" synthesis. "Intelligent Design Theory" seems to be more a reactionary reaction than a "synthesis", harkening back to "that old time religion" which seemingly millennia ago lost its true-faith connection with "revelation". IDT also seems to be correspondingly short of what would be useful revelation today, even though the great majority of the greatest scientists the world has yet known were all believers that their natural philosophy—or, more recently, science—was the devout study of the Handiwork of that "Big Intelligent Designer In The Sky". And ancient religions speak of our bodies as the visible vehicles evolved-designed for the invisible souls—us—who through them are enabled to try to learn to gain enlightenment, wisdom, and eventually salvation and "eternal life". So we can hope. (More on these topics in **Chapter 6, Newton's—and Science's—Great "… Oversights"; see p. 79.**)

But ironically, considering its persecution by the intolerants and intolerance of Religion, since the time of Newton (see comment on Newton and the wave nature of light, in **Section 6.4, Comparing Newton's Three Great… Oversights, p. 87**), and even before him, back even to the time of Aristotle (when Science *was already evolving*, even if not yet called science), Science itself has been evolving in this *same* direction of "Religious" intolerance of e.g. being questioned, unless that questioning remains within the "dogmatic" limits set by a powerful minority who are more concerned with the "status" of Science in society rather than with the "state" of Science's soul. Is this really appropriately an "eye for an eye", a "kill or be killed" situation, to provoke such a reaction on the part of supposedly more highly evolved—e.g. more wisely tolerant—Science?! Both Religion and Science need to do far *far* better in these things and ways.

Science, like religion, is going to continue to play an essential role in the future of human evolution. I love science no less because of its failures, even when those failures are on its own terms. But… "when wrong, to be put right" is absolutely essential to loving science the way I do. The same can also be said of religion, at times science's *seeming* "enemy", which I have come to appreciate and even love in much the same way I love science, and for much the same reasons: there is a light there that we need to learn to see with, to see the world—and ourselves—even if "through a glass, darkly". We tend to forget that religion was created/evolved not for people who don't need it, but for those people who *truly* do *need* it, e.g. those who all too frequently tend to fall in that satirical category of "he was a good man in the worst sense of the word; his were good intentions in the worst sense of the word; God damn all good men and their good intentions."

But, science is in its infancy, or rather still being literally conceived, even if religion seems to be its older brother, perhaps somewhat like Cain and Abel, or its older father, somewhat like Laius and Oedipus. And right beginnings, especially "when wrong, to be put right", are absolutely essential to its—*their*—and *our* future evolution. Science has been criticized on many grounds, especially in the modern "Science Wars" that have heated up in recent decades, but science has *never* really been publicly found and acknowledged (or at least publicly admitted) to fail on its own

terms, and fail so egregiously as it has here—excepting perhaps the "Newton Wave Wars" with its "Newtonian Inquisition" which extended well into the 19th Century (more on which later).

- ✓ I present "Newtonian Inquisition" as a deliberately (seemingly) exaggerated buzz-term, as would be the terms "Aristotelian Inquisition", "Ptolemaic Inquisition", or "Coulombian Inquisition", but a buzz-term that gives a distinctly clearer picture of some essential aspects of science over its entire history/evolution, which includes all too much of what is distinctly "Inquisition" in the far more well known sense. A clear example of this is the treatment Thomas Young received at the hands of "Newtonian Inquisitors" in the early 1800s when he tried to publish his work on interference experiments, even when he tried to insist that his own work really derived from Newton's, and that he was extending rather than destroying "Newtonian doctrine". (See I. Bernard Cohen's Preface to the revised Dover edition of 1979 of Newton's *Opticks*, p. xi.)

  Another example comes from a quote of the renowned physicist André-Marie Ampère (1775-1836), here taken (in context) from Alan Hirshfeld's biography of Michael Faraday, *The Electric Life of Michael Faraday*, Walker and Company Publishing, 2006, pp. 73-4. "While performing a demonstration before a science class, Danish physicist Hans Christian Oersted had noticed that an electrical current flowing in a wire moved a nearby compass needle. … After Oersted's announcement, physicist André-Marie Ampère lamented to a friend, 'You are quite right to say that it is inconceivable that for twenty years no one tried the action of the voltaic pile on a magnet. I believe, however, that I can assign a cause for this; it lies in Coulomb's hypothesis on the nature of magnetic action; this hypothesis was believed as though it were a fact [and] it rejected any idea of action between electricity and the so-called magnetic wires. This prohibition was such that when [physicist] M. Arago spoke of these new phenomena at the Institute, they were rejected . . . Every one decided that they were impossible.'"

  We often think of Science as being inherently more enlightened than Religion, but there are far too many of these examples of Inquisition-like asphyxiations of scientific exploration, advancement, and heedfulness in general.

This book will present a "quick, in depth look" at such a failure, not only of the science of Newton himself but of all scientists since, a failure squarely in science's own territory and on its own terms, an incredible failure, one that we can, with grace-saving and pedagogical humor, characterize as an "… oversight". It is hoped that when readers see how obvious this "… oversight" is in retrospect, that they will "wonder mightily" what else Newton and Lagrange, not to mention all the rest of us, have "… oversighted", and may still be "… oversighting". It is an especial hope that this book will also help to inspire the evolution of a dedication, not just within Science, but within our world community as a whole, to the ideal of "… when right to be kept right; when wrong to be put right."

DEDICATION

This book is especially dedicated to and intended for "stoutly gifted children of any age" ("be thou as little children…"), any gifts, those who are most likely to see **"Newton's Great… Oversight"** less as a "politically incorrect" "scientific heresy" and more as an **enormous and challenging opportunity**—silver, if not gold or platinum—for the future evolution of our science, of our history and philosophy of science, and especially of our science education, which should include quite a bit more history and philosophy… and quite a bit more of the science version of "comparative religion".

READERSHIP-AUDIENCE

The target readership for this book, as you should be able to guess by now, is "stoutly gifted children of any age", believers in "Science, right or wrong; when right, to be kept right; *when wrong, to be put right*" who will find it a *challenging opportunity for the future evolution of science.* This mainstream audience—primarily young and scientifically and philosophically sophisticated—will include Professional Scientists, Science Educators, "Popular Scientists", "Newton Lovers", "Popular Philosophers of Science" and "Popular Science Philosophers", "Science Warriors"—and perhaps now "Popular Science Warriors"—and in general all who have serious concerns about the state and future of science and science education in the world, all of whom will find this ebook a "must read" and "must discuss".

This book will appeal to **all who love science**, **principally science promotion and science education, "popular science"** and its philosophical counterparts, **"popular science philosophy"** and the **"popular philosophy of science"**, especially anything having to do with that **fascinating**—in retrospect, from three centuries worth of safe distance—**Isaac Newton**, one of the most ever-popular figures in our modern popular science, and even more especially when it becomes necessary to correct an interesting scientific "… oversight" made by that same Newton, and by every scientist and science educator since.

Many will be drawn to these **New Controversial Findings** concerning **Newton's theory of gravity (and Newton himself) and Galileo's falling bodies**, and the **physics and astronomy** of those strangely behaving **Trojan asteroids** (which can be seen and even studied with good amateur telescopes) with their strange "tadpole" and "horseshoe" orbits around the equilateral Lagrangian points L4 and L5… **science at its most fascinating!**

It is expected that the **Controversial Nature** of the idea of the great Isaac Newton making a serious **"… oversight"—*one that we all still make*—**may *at first* make potential readers feel shy about possible "scientific heresy" or "political incorrectness", both all too common psychological reactions. But ultimately—and probably quickly—it will be seen that the situation is far more fascinating than threatening, indeed that it will open up a **goldmine of opportunities** for further analysis and commentary (and publishing rather than perishing). Then many will want to explore the **"Brave New World"** of this and other **scientific "… oversights"** and what they mean for **the future evolution of our science and its philosophy, and especially for our science education**.

"Science Warriors": On the other hand, this controversial idea of **science failing on its own terms** and related philosophical analyses—controversial to the point of being **"politically incorrect"** and even polemically so—will *itself* attract many readers who are rightly concerned about the ever more crucial role science is playing in society, and about the possible dangers to society when science fails so… "interestingly". The so-called "**Science Wars**" normally deal with (most often at least partially valid) criticisms of science on grounds that many scientists reject as "unscientific", such as sociological, cultural, feminist, religious, psychological, linguistic, anthropological, and on and on. But this ebook presents a *unique* (in modern times) and *uniquely clear* case of **science failing**, and failing egregiously, not on some non-scientist's "unscientific" terms, but **on science's *own* terms and on its *own* turf, *and failing for 300++ years, and still failing even today*. Chapter 6, Newton's—and Science's—Great "… Oversights"** (see **p. 79**), gives a brief historical-psychological analysis of this oversight, with a long and *atypical* philosophy of science style analysis and critique of science, on the sources of failures in science such as we find here, and should devoutly wish to stoutly avoid in the future.

Dear Reader: the **science** in this book is as **simple** and **basic** as it ever gets in serious science, and roughly the standard level for serious popular science readers. There is **mathematics** for those who will rightly want to verify the science in that way, but it is likewise only relatively **simple algebra and trigonometry** (and implicitly a bit of vectors, since the forces act in different directions), and some basic common sense mathematical logic. The basic gravitational mechanics involved does not even need calculus, let alone the partial differential equations, calculus of variations and perturbation theory that the world famous mathematician Lagrange used (and largely developed). So the math is not even the most difficult sometimes found today in popular science. And it sure beats the heck out of Lagrange's arcane perturbation theory, which only a handful of people in the world know truly well.

This book should also appeal to **professional scientists, especially physicists,** who want to make sure that what is presented here, despite being controversial, is true science, a true "… oversight", and not just some "New Age Nincompoopadoodle", or the ravings of a **"Sulky Science Warrior"** who "has a thing against science and just wants to find fault with it".

And when the controversy has cooled, and it will fairly quickly, this book should find a place as **supplemental material in science education**, including **physics** and **astronomy** classes, **philosophy** and **history of science**, **self-study**, and sociology, psychology and other Science and Technology Studies (STS) classes and seminars dealing with the **"Science Wars"**, which are truly important multifaceted inter-evolving phenomena relating to **the future evolution of our science and our science education**.

[This page intentionally left blank.]

1 INTRODUCTION

1.1 VARIOUS WARNINGS:

Reminder: the "target" readership for this book *is* "stoutly gifted children of any age", believers—or potential believers—in:

> **"Science, right or wrong;**
> **when right, to be kept right;**
> ***when wrong, to be put right*".**

Its objective is to proffer certain *challenging opportunities for the future evolution of science.*

My choice of target readership—not just young and scientifically and philosophically sophisticated, but conjointly sympathetically indulgent of quirkiness and eccentricity, in themselves as well as in others—allows me to indulge my personal style, which is most often unconventionally informal, often leavened with humor… or attempts at some such. I am in fact proud of the English eccentricity I inherited with my DNA, albeit quite humbly so. At least for a British reader, this will help explain the final Science and Technology Studies-style "Science Wars" philosophical critique that often verges on the Russellianly obstreperous, at times a no holds barred diatribe, at times maundering on seemingly endlessly, distinctly *not* for the faint of heart.

Anyone who reads this book straight through may notice repetition of important points, including those made just above in this section. I am currently rationalizing this on the basis that very few will bother to read the book straight through, and they would be significantly more bothered by the lack of essential information needed at the salient points than those who do happen to read it straight through would be bothered by the repetition. I am further rationalizing it on the basis that repetition of essential points is a time-honored didactic/pedagogical device.

I similarly rationalize giving fullish-style references in the text rather than the terse kind: e.g. I prefer "Thomas Kuhn, *The Structure of Scientific Revolutions*, any edition, p. 62" to "[Kuh1970, p. 62]" or "Kuhn [1970, p. 62]" because the former gives more and friendlier information where it is needed. I personally get tired of always having to look in the Reference section to see what publication is being referred to get a further clue to what is being communicated by the author, often obscure enough as it is.

I also proffer profuse personal apologies for the inexcusable lack of a good Index (at this time).

1.2 Of Apples and "… Oversights"

When that Apocryphal Apple Fell and hit young Isaac on his Mythical Head, his great "Ah-Ha!" was:

- **Newton's Great Insight into Gravity:**
  not only does the Apple fall toward the mass of the Earth,
  but *the Earth also falls toward the mass of the Apple*.

For historical purists, 1666 is the year attributed to Newton's conceiving the idea that the Earth's gravity extended all the way to the Moon, diminishing in proportion to the square of the distance. Robert Hooke (1635-1702)—Newton's favorite, if not greatest, arch-nemesis, his Moriarty—also claimed credit for the inverse square idea, probably justifiably, and also publicly accused Newton of plagiarism in that matter (and others). (See "Seeing Further, The Legacy of Robert Hooke", Kathy Miles and Charles F. Peters II, at http://starryskies.com/articles/spec/hooks.html for more on the Hooke-Newton wars.)

- ✓ Charging a genius like Newton, someone who has quicker and greater insights into so many things, with plagiarism is a questionable proceeding. Newton was the kind of person who might have thought of "the idea" long since along with a zillion other ideas, but also happened to later be significantly sparked by someone else's presenting "the idea" and inspired to quickly take "the idea" yet further. Newton's personality was such that he was not easily led to publicly acknowledge—and perhaps not even privately remember—the sparking contributions of others, however significant. And to be charged—even privately, but especially publicly, as Hooke did—with *any* fault at all was anathema to him. (See Richard Westfall's excellent *Never at Rest, A Biography of Isaac Newton*.)

But, in this particular instance, we are dealing with the Apocrypha if not the Myth of the Apple and the Fall.

How, then, did Newton fail to note that his insight meant that a heavier apple would fall to Earth faster than a lighter one (in the separate release case, we need to note)?! The equation is as just about as simple as it gets in physics: the acceleration of the Heavier Apple toward the Earth plus the acceleration of the Earth toward the Heavier Apple is greater than the acceleration of the Lighter Apple toward the Earth plus the acceleration of the Earth toward the Lighter apple. The *instantaneously* ***initial*** (gedanken) accelerations of the Heavier Apple and Lighter Apple are *equal* in Newton's absolute space-time frame, given that all three start with zero velocity in that frame, leaving out the far lesser but obviously unequal accelerations of the Earth toward each of the Heavier and Lighter Apples.

This failure to note a simple, obvious, but essential consequence of his own theory of gravity—in the context of his time and the Galilean belief from decades earlier that lighter and heavier bodies fall at precisely the same rate—is "Newton's Great… Oversight".

1.3 "… Oversights"

There is an old and somewhat satirical aphorism from the world of chess:

- ➢ **When a beginner gives away his queen, it's a blunder.**
  When a grandmaster gives away his queen, it's an… oversight.

Since ancient times humor has been used as a mnemonic and a teaching device, in addition to "comic relief". Shakespeare is a constant reminder of how all important that last can be. The ellipsis "…" has here been added to the old adage as a modest didactic device (the technical term is "gimmick") to further all these.

1.4 "Wave Auf Wiedersehen!"

Although it is only rarely recalled or remarked on, (as noted in the **PROLOGUE, p. 12**) Newton is already known in scientific circles to have made certain "great" scientific oversights. It is commonly known, for example, that he overlooked the inspired and inspiring $\mathbf{d}f(x)/\mathbf{d}x$ notation that Leibniz developed in the same years that Newton was privately developing his own variant of "the calculus". He did not adopt it even after he came to know of Leibniz's contributions in that area, probably partly because of their intense rivalry for priority in the matter, and partly because he had a morbid fear of the concept of a numerical "infinitesimal", exacerbated by the general feeling among intellectuals of Newton's day that "infinitesimals" were a "heresy" to be shunned.

- ✓ It's interesting to note that people at that time usually referred to these ultimately small entities as "indivisibles", as if they were anticipating some kind of "atomic" or "quantum" level of existence "down close to zero", a distinctly different concept from being able to get "arbitrarily close to zero".

And even today it seems incomprehensible that Newton, "the greatest scientist who ever lived", completely rejected any "wave" concept of light in favor of his own "corpuscular" theory of light. (Well, he temporarily espoused an "undulatory nature" for light, but later divorced himself from it. All this will get its own "quick, in depth look" in **Section 6.4, Comparing Newton's Three Great… Oversights, p. 87**.) Neither of these is, however, the "Great… Oversight" to be presented and explored in this book.

Here we are going to look at an "… oversight" that *not only Newton, but every other scientist since has made to this day* (as of this writing, February 2010), that Newton's own theory of gravity predicts that lighter and heavier bodies *must* fall at *different* rates—with only special exceptions. (The reader who seriously loves science might wish to note psychological reactions—her or his own, and those of others—to this statement, for later contemplation.)

The story, ever repeated though apocryphal, tells us that Newton discovered his law of gravity while sitting under an apple tree when an apple fell on his head. "The apple falls toward the Earth, but *the Earth also falls toward the apple!*" It had come to him that the apple was attracted to the Earth due to the mass of the Earth, but that the Earth must also be attracted to—and will fall toward—the apple due to the mass of the apple. His insight, in more detail, eventually came to be that every material body attracted every other body in proportion to the "mass" (considered a metaphysical concept by many eminent physicists as recently as the early 20th Century) of the first body, and was attracted to every other body in proportion to the mass of that other body. (We will get to the bit about the inverse square relationship with distance later.)

Newton then made probably the greatest oversight—in the strict sense of the word—of his illustrious career. He *failed* to follow his Great Insight into gravity to its logical conclusion:

- ➢ **Newton's Great… Oversight:**
  Galileo was wrong:
  Lighter and heavier bodies do *not* generally fall at the same rate.

Lighter and heavier bodies do *not*—in either theoretical or physical fact—fall at the same rate, *except* in very restricted circumstances. If Galileo had released them both—in separate trials—from the top of the Tower of Pisa, the Earth would have fallen toward the heavier apple faster, so the heavier apple would have fallen toward the Earth slightly *faster* than the lighter apple! I.e. the *heavier* apple will fall faster than will the lighter apple, as measured in the *reference frame of the Earth*. We can also say the Earth falls faster toward the heavier apple, as measured in the *reference frame of the heavier apple*, than it does toward the lighter apple, as measured in the *reference frame of the lighter apple*—to use *terminology* from relativity.

So Galileo, by Newton's day almost a demi-god—or at least an Apostle or a Saint—of the then newly burgeoning Renaissance stage of "natural philosophy" that was busy evolving into what we now call "modern science", was… wrong, *scientifically wrong*, when he held that lighter and heavier bodies fall at precisely the same rate, even though Galileo was right as an *engineering approximation* (emphasis on *approximation*), one that is still close enough for many useful things. (Relatedly, we continuingly overlook that the ballistic flight-path of the traditional cannonball is

not really a parabola, but rather—per Newton—the section of an ellipse, still abstracting out and ignoring those ever-present "other factors".)

- **Today, well into the beginning of the 21st century, scientists and educators *still* teach and still *believe* that lighter and heavier bodies—per Galileo—fall at precisely the same rate. But the truth is *far* more fascinating!**
- **Abstract Reasoning:** you keep abstracting out the terms associated with the "other factors" until the product of the factors that are left shows the reasoned conclusion: e.g. that lighter and heavier bodies fall at the same rate. Much the same is done in general when one lies with statistics.

The situation gets especially fascinating when we look at the case of the *simultaneous* release of the 2… uh, apples. The extra complexity of a 3 apple system—gedankening the Earth as a rather large apple—gives us a synergy that was lacking in the separate release case:

- A trivial and *still* overlooked consequence of Newton's laws is that, because of their asymmetric gravitational interactions, when released simultaneously some distance apart but at the same distance from e.g. the Earth (so that all 3 form an isosceles triangle), lighter and heavier bodies do *not* fall at *precisely* the same rate, with only one exception: if there are precisely 3 bodies involved, i.e. including the Earth, the lighter and heavier bodies *will* fall at precisely the same rate—Newton-theoretically—when the 3 masses are at the vertices of an equilateral triangle, i.e. when each of the 3 bodies occupies a Trojan point—i.e. a Lagrangian point L4 or L5 (see **Figure 4: The Lagrangian Points L1-L5, p. 139**)—with respect to the other 2 bodies.

Newton, although he has been called the "last of the magicians" and the last wonder-child to whom the Magi could do sincere and appropriate homage" (by John Maynard Keynes in his *Newton, the Man* (see http://www-groups.dcs.st-and.ac.uk/~history/Extras/Keynes_Newton.html), partly because of his deep interest in alchemy, was perhaps the greatest scientist who ever lived. He was also a fanatic who would never give up until he was able to account for what to others might seem like the minutest of discrepancies. (He seemed to be happy with 1 part in 10^8.) Newton, of all people, should *never* have overlooked that his *own* theory of gravity predicts the (mostly) non-zero falling rate difference between heavier and lighter bodies, that this difference implies the existence of Trojan points, and that it is essential to the gravitational dynamics of these Trojan points and their associated Trojan asteroids, which are now known to move in "tadpole" and "horseshoe" shaped orbits "around" those points. (Actually, the orbits can get quite a bit more complex than that suggests.)

- If Sir Isaac—or even his lesser contemporaries—had only questioned the modern scientific dogma of their day, and had been willing to sacrifice that "Sacred Cow", he—or they—could easily have discovered Trojan points almost a century before Lagrange.

It is this "… oversight" that we will explore, mainly from the standpoint of how easily it could have been avoided. We will look at the historical precursors, and then, unlike most popular science/philosophy expositions, we will explore in rather more detail the quite simple mathematics and other formal reasoning of this quite simple approach to the seemingly scientifically exotic Trojan points of Lagrange. But the

mathematics and reasoning are in fact so simple that even one of Newton's lesser contemporaries would have found them accessible and compelling, perhaps could—should—even have discovered them. Genius is/was not needed, certainly not genius such as Newton's. Perturbation theory is not needed, nor the calculus of variations (of which Lagrange is also considered the founder), nor partial differential equations, nor even basic calculus. Only basic trigonometry, basic algebra and an elementary understanding of vectors are needed, such as one might find among science students in modern high schools, along with the ability to do simple reasoning about physical reality. After seeing how simple the physics actually is—and would have been even in Newton's day—we will look at some of the general factors and particular influences that may have brought about *Newton's Great… Oversight*.

[This page intentionally left blank.]

2 HISTORY

Before getting to the details of the physics and the equations, and the further analysis, critique and commentary, we will take a quick look at the history of the falling rate difference contretemps. Be prepared for frequent references to Newton's, and other "… oversights". (See **Chapter 6, Newton's—and Science's—Great "… Oversights", p. 79**.)

2.1 Aristotle

Aristotle (384-322 BCE) was a *fantastically* famous Greek philosopher-scientist (and perhaps still is), known among *many* other things for being a student of Plato and the tutor of a young boy named Alexander, later known as the Great. Aristotle was so influential that, although a "pagan", he was solidly adopted into the pantheon and dogma of the Church as an unquestionable authority (almost the only kind there is, though some are more unquestionable than others; Aristotle was *very* high up in that pecking order, all the more solidly because he was solidly dead by that time). Until quite recently (the last couple hundred years or so) it was considered *serious heresy* to question his teachings. (Sir Isaac was "knighted", but others found themselves "ignited".) Copernicus (1473-1543), for example, questioned Ptolemy (ca. 100-178), but would not—or perhaps felt he *could* not—question Aristotle.

Many people know that—among *many* other things—Aristotle held that heavier bodies fall faster than lighter ones (actually he said something more like larger bodies fall faster than small ones; it's difficult to tell if he was thinking of anything resembling air resistance). If he had stopped there he would have been at least partly scientifically correct since the Earth and the heavier body will fall together faster, not much faster, but both theoretically and actually faster—if released in separate trials.

But Aristotle also held that a body would achieve its final velocity at the instant of release, as if "impulses" exist (in the sense of physics: an impulse is work or energy that is transferred "instantaneously", and thus not analyzable as Work = Force x Distance), but not "fields" and "action at a distance". I.e. it was as if he thought of gravity as supplying an instantaneous impulse that acted only at the instant the body was released, and that after that the body continued moving with no forces acting on it, in uniform, un-accelerated motion.

Actually, that is not quite correct, either: Aristotle held that "forces" would keep the body in motion until the forces ceased and the motion also ceased, a different concept of "forces" and "motion" than Newton's, to be sure. This should all sound a *little* familiar if you have studied ancient philosophy, but it needs only a very crude subjective experience of falling bodies here on Earth to find untrue. (Visitors from other dimensions, however, could easily find us boringly provincial. Also, astronomers have begun to question whether gravity does interestingly different things in the vast intergalactic regions of space-time.)

Amazingly influential even today at the beginning of the third millennium of the Common Era, Aristotle was still a quasi-deity 1900 years after his death, when a certain Galileo started making trouble, questioning the established modern science, not just of Aristotle's time, but of his *own* time.

2.2 Galileo

It isn't until Galileo Galilei (1564-1642) that we have well known historical records of anyone not only disputing Aristotle's position, but performing experiments to verify that heavier bodies do *not* fall faster than lighter ones, rather that they fall at the same rate—or to be more precise, observably the same rate under the conditions of Galileo's

experiments (by the standards of that day, but not by 20th Century standards). Galileo left records of rolling different weight balls down an inclined plane, which allows for more accurate timing than dropping them from the Tower of Pisa even if it introduces other considerations such as rolling friction and angular momentum. He experimentally found them to roll at the same rate, and he *hypothesized* that they *fall* at the same rate—*precisely* the same rate.

Galileo also left records of a gedanken experiment—thought experiments were popular long before Einstein—that asked one to imagine dropping 2 different weights with a chain attaching them to each other, and then to think that the chain does not go taut if they are released at the same instant—and of course to compare this with their own, i.e. the readers', experience of reality. Then one was to imagine dropping them with no chain connecting them and to think that they *must* fall at the *same* rate, with the presence or absence of the chain assumed to not affect the falling rate. (This was also part of the basis of his same-falling-rate hypothesis.) The romantic but almost certainly apocryphal story we all know and love has Galileo dropping 2 such bodies—I like to think of them as Newtonian apples—from the top of the Tower of Pisa.

- ✓ Some of you have already started complaining that in 1586 Simon Stevin, the Flemish engineer, beat out Galileo by several years, in fact actually dropping lighter and heavier bodies from a church tower in Delft. (See the very quick bio "Simon Stevin, Flemish tutor of a Dutch Prince" which can be found at http://users.ugent.be/~gvdbergh/files/publatex/stevinoe.html)
  Another fascinating Renaissance personage!

It is surprising how many people today—especially scientists, and even physicists and educators—still believe and still teach that Galileo was scientifically correct when he hypothesized that lighter and heavier bodies fall at (precisely) the same rate. The truth—as usual—is far more fascinating than we currently know it to be. This is not to say that Aristotle was completely correct, that heavier bodies *invariably* fall faster than lighter bodies relative to e.g. the Earth, but they do much of the time. This can be demonstrated mathematically—and astronomically—even when deliberately "abstracting out", i.e. neglecting and/or ignoring, other real world effects such as viscosity (for purposes of simplification, etc, as is usual in science), i.e. neglecting everything but Newtonian gravity and mechanics applied to 3 point masses. With regard to falling rate differences of lighter and heavier bodies, scientists have neglected since the days of Newton that the approximations involved hold—or are accurately extrapolateable—only within limits, often quite strict ones. This is an all too common failing in science that will be commented on again later.

The far more interesting failing of Aristotle's theory—thesis might be better; no *hypo*-thesis for *this* guy—is one shared with Galileo's. If the 2 bodies are released at the same instant, the lighter one accelerates toward the heavier one faster than the heavier one accelerates toward the lighter one, and both these accelerations have vector components in the direction of the center of mass of the Earth. Paradoxically, whenever they are closer together than 60 degrees, as they would be if Galileo had actually dropped them simultaneously from the Tower of Pisa, the *lighter* body falls faster! Ironically, Aristotle and Galileo were *both* scientifically wrong!

The falling rate difference, although *usually* very small, may *perhaps* soon be directly detectable near the Earth's surface by tomorrow's laser equipment, but in any case it is readily detectable astronomically in certain cases of orbiting bodies. The prolonged orbital fall gives them enough time for the falling rate difference to affect their orbital positions in a way visible to even the telescopes of the 1800s (well, of 1906), when astronomers finally discovered the "Trojan Planets" Lagrange had

predicted over a hundred years earlier. (Or was it 1904? For more of the fascinating history of the discovery of the Trojan asteroids see
http://www.cfa.harvard.edu/iau/pressinfo/TheFirstTrojanObs.html)

One of Galileo's main concerns was to refute Aristotle, who had established as scientific dogma, among other things, that heavier bodies fall faster than lighter ones. This kind of thing—refutation—is all too often considered dangerous, and Galileo almost lost his life—tenure of sorts—to the Inquisition for questioning the still accepted modern scientific dogma of his day. (Remember: Ptolemy had not yet been traded in for Copernicus by the Keepers of the Flame of Modern Science, in particular Galileo's fellow modern scientists, the ones who refused to look through his telescope, who were about to be traded in themselves.) For example, Galileo's *Dialogue Concerning the Two Chief Systems of the World* (*Dialogo sopra i due massimi sistemi del mondo*), published in 1632, was put on the Index by the Church, where it remained until 1822.

- ✓ By way of ecumenism: just a few years ago—in 1993—after a special Vatican commission finished its investigation of the matter, Pope John Paul II issued a reassessment of the famous 1633 case. He said that Galileo Galilei was unjustly condemned by the Roman Catholic Church for promoting a Copernican cosmology. He did not, however, condemn that condemnation, nor associated censorship, nor the Inquisition itself (by whatever name it happens to be known), nor censorship in general.

2.3 Newton

By the late 1600s, at least no later than the publication of his *Principia* in 1687, Isaac Newton (1642-1727) had formulated his still famous theory concerning gravity.

- ✓ Although he probably had immense help—e.g. in arriving at the inverse square law of gravity—from Robert Hooke, Newton himself was perennially "disinclined" to give credit where it was due. Worse, he had come to not merely despise, but to loathe Hooke. Not merely denying him due credit, Newton, made president of the Royal Society in 1703, the year of Hooke's death, is reputed to have "magically disappeared" Hooke's scientific papers and equipment, and even to have quietly refused a great deal of money that Hooke had tried to bequeath (unfortunately without a will to back it up) to the Royal Society. (Again, see "Seeing Further, The Legacy of Robert Hooke", Kathy Miles and Charles F. Peters II, at http://starryskies.com/articles/spec/hooks.html for more on the Hooke-Newton wars.) Newton, knighted in 1706, 3 years after Hooke's death, for his fantastic success in reforming the coinage (which greatly helped the British Empire take root around the world), also succeeded handily in both outliving him and outshining him.

Newton's great insight—or rather, among the many great insights attributed to him—was the concept that the same gravity that attracted (the mass of) the apple to (the mass of) the Earth also attracted (the mass of) the Earth to (the mass of) the apple, and *even* (the mass of each of) 2 apples to each other.

Although he made it explicit in his theory that (the mass of) the Earth was gravitationally attracted to *any* other mass, and that the Earth fell through space as did all other masses, Newton slipped up scientifically with regard to an essential—and very important—consequence of his theory. It is almost totally inexplicable, but Sir Isaac did not use his theory to carefully re-analyze the well-known, paradigmatic (again in the sense of Thomas Kuhn; see his *The Structure of Scientific Revolutions*, any edition), same-falling-rate finding of Galileo. It is also difficult to understand that

neither did his lesser contemporaries—nor have any physicists or astronomers from their day to almost a full decade into the beginning of the 21st Century—since it turns out that only reasonably simple algebra and trigonometry are needed to show not only that a non-zero Newton-theoretical falling rate difference theoretically exists, but to also (at least begin to) predict the existence of Trojan asteroids orbiting what are now known as Lagrangian points L4 and L5. (See **Figure 4: The Lagrangian Points L1-L5, p. 139**.)

Galileo's falling body problem is essentially a tractable example, even a very simple one, of the classically intractable *n*-body problem for inverse square fields ($n \geq 3$), and this has been ignored since the time of Newton's *Principia*. Newton's *Philosophiae naturalis principia mathematica* was only first published in 1687, at the insistence of the English astronomer Halley, who also paid for the printing, though Newton had conceived his theory of gravity many years earlier. Newton's *Principia* soon became *very* influential, even though many kept trying to prove Newton wrong about his various theories and hypotheses for over a century after its publication.

With everything else he did so brilliantly and insightfully, it is *excessively* strange that Newton did not question Galileo's finding, *so* much so that it bears a *great* deal of repetition. If he had, he certainly would have noticed that it was scientifically incorrect, *especially* in the context of celestial mechanics, with which he was famously involved. After all, it was Newton who had the brilliant insight that *any* mass exerts a non-zero gravitational force on *any* other mass, including on the Earth. He, at least, should have noticed too the asymmetry of the masses of Galileo's 2 falling bodies and guessed that the falling rate difference must not only be non-zero, but noticeable if the bodies were in orbit. If he had, the credit for the theoretical discovery of the Trojan points and the planets or asteroids that potentially inhabit them would almost certainly have gone to *Newton*, and not to Lagrange.

2.4 Lagrange and His "Trojan Planets"

Joseph Louis Lagrange (1736-1813) was a famous Italian or French mathematician, depending on who you ask. He was born and baptized Giuseppe Lodovico Lagrangia in Turin, Italy, where he lived till he was 27, and became a well-known mathematician. He then went to France for almost 2 years, after which he lived mostly in Germany and, later, France, again. He perhaps did most of his important mathematics in France, with Germany a close second.

Among many other important accomplishments, Lagrange is credited with developing perturbation theory to solve special subclasses of the generalized, gravitational *n*-body problem for masses in an inverse square field. Perturbation theory is not considered simple even by today's standards, even for professional physicists, astronomers, *or* mathematicians. (It starts with partial differential equations and the calculus of variations, which Lagrange also pioneered, considered essential in physics but difficult to master except by an elite few, and makes them look simple by comparison.)

It is important to NOTE that, at first glance to the uninitiated, Lagrange seems to be solving a 3-body problem, but since he considered 1 of the 3 bodies to be "infinitesimal", he was actually solving a special 2-body problem, or perhaps a $2 \pm \varepsilon$ ("fractal"?) body problem. Well, thanks to Newton, i.e. before Lagrange, it was known that 2 spherical (uniformly dense) bodies follow conic sections with their common center of mass at a focus, and also that with respect to the other (as the origin in a coordinate system) each will follow a conic section. The laws of planetary motion proffered by Kepler (1571-1630) in 1609 had placed the (center of the) Sun at the

focus (well, one of them) of the elliptical orbits of the planets, but later that century, Newton's law of gravity (first *published* in 1687) showed that it must be instead the center of mass of the 2 bodies at the focus (well, one of the 2 foci) of the ellipse. Using his perturbation theory, and with some inspiring success, Lagrange added a ghostly "infinitesimal" 3rd body to the picture. (See **Section 3.3, "Infinitesimals" and Levels of Approximation, p. 41**.)

In 1772 Lagrange published a memoir predicting the potential existence of what he called "Trojan Planets" in the orbit of Jupiter, but leading/following Jupiter by ± 60°. In fact Lagrange found that all "homographic solutions" (see definition in the APPENDIX, p. 129) for 3 non-collinear bodies are equilateral triangles, and that together with the 3 distinct collinear configurations that he also found, they comprise all homographic solutions for 3 bodies. His theory also predicted that such a system—i.e. the 3 bodies in an equilateral triangle—will remain in a "stable equilibrium", within limits, even when "perturbed" by other forces such as the gravitational effects of other planets—thus the name "perturbation theory".

It must be noted, however, that Lagrange's concept of "stable equilibrium" allows the 3 bodies to diverge quite greatly from an equilateral triangle. Modern astronomy's concept allows even more. This allowed divergence is so great that both "tadpole" and "horseshoe" orbits can be considered regions of that "stability". (See **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, **Chapter 4, Trojan Points and Their Tadpole and Horseshoe Orbits, p. 67**, and **Figure 3: The 3 Masses and 2 of Their Centers of Mass, p. 138**.)

It was a limitation of Lagrange's theory, however, that 1 of the 3 bodies had to be effectively "infinitesimal" (more on which later). In Lagrange's day, if one did not make this simplifying assumption, then the problem reverted to the usual intractability of the more general 3-body problem. Lagrange also showed (within the limits of his assumptions) that, for the Trojan points (i.e. L4 and L5), 1 of the 2 *non*-infinitesimal bodies had to be much smaller than the other (for "stability"; more on this later). With modern computers, this limitation is no longer necessary computationally, and it is certainly not desirable theoretically, although advancing theory to take it into account may still prove quite difficult. The cumulative effect over time of the actual "non-infinitesimality" of the third mass may lead to a substantial divergence of theory and reality.

- ✓ We should also note a different kind of limitation of Lagrange's theory. It is noticeably more arcane than the calculus of variations (also of Lagrange) and the partial differential equations on which it is based. Probably only a few astronomers and/or physicists and/or mathematicians in the world feel comfortably familiar with it. The results to be shown here, however, do not even require *basic* calculus, only basic algebra and trigonometry. It will be pointed out *more than once* that even Newton's *lesser* contemporaries were guilty of overlooking such an obvious set of results.

More than a century after Lagrange, in February of 1906 (with some possibility of historical error for that date), the astronomer Max Wolf finally proved that Lagrange was correct 134 years earlier by discovering the first Trojan asteroid, 588 Achilles, in the leading Trojan point L4 "tadpole" of Jupiter. (See **Section 5.2, Trojan Point Astronomy in the 20th and Early 21st Century, p. 71**.) His discovery is also the first historically verifiable scientific observation that corroborates the falling rate difference of lighter and heavier bodies predicted by Newton's laws, even if this verification is only appreciated in retrospect, for the first time here in this book.

One more fact is historically interesting here. It wasn't until about 20 or 30 years after the death of Lagrange that "science" got its modern name. For centuries if not millennia, "it"—as "it" was evolving—was called "natural philosophy", because it was considered to be the philosophy of Nature. Actually, that was after natural philosophy had branched off from the far more general "philosophy"—"love of wisdom" (ask Hippasus, one of the first official benefactors of some such)—which had its origins in Greece. Although one would think there would be a historical-evolutionary connection between ancient Greek philosophy and Eastern philosophy, thousands of years older yet, if only because the Sanskrit language is so closely related to Greek and Latin, none has ever been publicly advertised.

2.5 Poincaré's Further Work on the 3-Body Problem, and Chaos Theory

A quick digression: much more work has been done since the time of Lagrange in celestial mechanics and finding solutions to the 3-body problem and later the generalized *n*-body problem. In particular the name Henri Poincaré stands out. Jules Henri Poincaré (1854-1912) was a famous French mathematician. He is one of a handful of the greatest mathematicians ever. He was perhaps the last mathematician to "master" all the then known fields of mathematics. Today there are way too many fields of mathematics for even a genius like Newton to become well acquainted with them all, let alone study them all deeply.

Poincaré's work in *partially* solving the (restricted) 3-body, interestingly, has today become quite important. He discovered that the trajectories of some systems would rapidly diverge even though their initial conditions were closely approximate. This is an essential result that has not been truly appreciated in our scientific, mathematical, logical and philosophical communities. He also discovered that there were completely (as opposed to quasi-) periodic solutions for 3 bodies that would today be called "chaotic". His was the first historically known work to make such huge inroads into what we today know as "chaos" that it is now considered the foundation for today's "deterministic chaos theory". We will return to this in **Section 7.2, A Seeming Digression: Poincaré's Chaos and "Approximation"**.

2.6 Einstein and His "Relativity"

Another quick digression: Newton and Einstein are the two biggest names in our modern (Western) science.

- We can say that Newton was/is its King Arthur,
  but Einstein is without a doubt the Elvis of our modern science.

Even with his sour personality and paranoid schizophrenia, Newton achieved fantastic recognition and acclaim by the end of his life, and even extensive worship after it. (See the quote by Alexander Pope on **p. 86**.) But Einstein gained more world recognition, not only for himself, but for science, and perhaps more importantly for the possibility—even the necessity—of combining it with a warmly compassionate humanism, and even religion, philosophy's and science's older sibling. Albert Einstein means physics to most people today in a way that Newton may never do (again?). People still find themselves infatuated with Einstein the person, Einstein the human being, in a way that they never have felt about "Sir Isaac" from centuries earlier.

This is a lead in to mentioning that there is a question: if lighter and heavier bodies "fall" at different rates, especially in the case when they are released in separate trials, what will this mean for Einstein's theory since relativity requires that lighter and heavier "test particles" always "accelerate" at the same rate? It all hinges on the

difference between “fall” (“relative” to other bodies) and “accelerate” (“relative” to an “absolute” space-time frame of reference, much like Newton’s).

Relativity requires the gedanken existence of Lorentz frames of reference, i.e. of “inertial” frames of reference that move in “uniform” or “unaccelerated” motion, so that the acceleration does not induce forces that mimic gravity. Somewhat embarrassingly, scientists tend to overlook that this uniform-unaccelerated motion must be with mathematical reference to an implicit but “absolute” Newtonian-style frame of reference. Although Earth is considered to be a Lorentz frame to a good approximation for many classical purposes, this approximation does not extend to the difference in falling rates of lighter and heavier bodies in the sense of Galileo (extended to many more decimal places of accuracy, together with Newton and, more implicitly, Lagrange). In a “gedanken real world situation”, lighter and heavier test particles, released from the “same point” in separate trials, will accelerate at the “same rate” with respect to the “same frame of reference”, and one that is not accelerating with respect to a Newtonian-style absolute frame of reference at the *precise* instant of release ($t = 0$). But thereafter— $t > 0$, even “infinitesimally greater than zero”—they will accelerate at *different* rates because they have caused all the other bodies/masses to accelerate and reposition themselves *differently*, yielding a *different* gravity “landscape” in which they are accelerating with correspondingly *different* accelerations, even the “absolute” accelerations that relativity requires to be strictly equal, at least at time $t = 0$. So the question seems to resolve itself in favor of relativity… at *precisely* time $t = 0$, and no other.

[This page intentionally left blank.]

3 NEWTON'S GRAVITY AND LAGRANGE'S TROJAN POINTS

3.1 Newton's Theory of Gravity

Newton's theory of gravity, which he had seemingly developed long before he published it in 1687 (there are "hints" that he left "retroactive notes" of his earlier work on the subject among his actual notes of earlier work on everything; rats! I lost the precise reference, so try Westfall's wonderful but longish bio) in his *Principia* (see **Section 3.4, Some Basic Equations and Some Simple Equations, p. 44**), included as fundamentals that:

- *all* masses have a *non-zero* gravitational effect on *all* other masses; i.e. each mass exerts a force on each other mass that is proportional to each of the 2 masses (and thus their product) and inversely proportional to the square of the distance between them
- *all* masses fall in/through space "toward each other" (though in general *not* toward the center of mass of all the other bodies, even assuming an infinite speed of gravity)

The first of these means that, in theory, no mass can truly theoretically be classed as "infinitesimal", i.e. as not gravitationally attracting other bodies (more, below). The second means that the concept of "falling" *must* be other-body relative, e.g. Earth-relative. For Newton's theory, there exists an absolute space (-time) frame of reference for all motion, but all masses are busy accelerating around in that frame of reference because they experience (at the very least) gravitational forces from all the other masses that are also all accelerating around. Theoretically, *no* mass can *in our physical-material reality* experience "uniform, un-accelerated motion", though we at times think we approximate it. We may not notice these other accelerations because our local Earth gravity is much greater, but they are there, even with falling apples.

Using Newton's laws, one can give simple algebraic-trigonometric expressions for the initial instantaneous accelerations of both the lighter and the heavier falling bodies of Galileo, as well as that of the Earth, toward each other. To get the falling rate difference of the lighter and heavier bodies we can, in modern terms, sum the vector components of accelerations of the lighter body toward the Earth and of the Earth toward the lighter body, likewise sum those of the heavier body and the Earth, and take the difference. Although they did not have vector arithmetic, a variant of this approach would have been simple enough even for Newton's lesser contemporaries.

Here we deal only with the case that the lighter and heavier bodies are equidistant from the 3rd, Earth-like body (or, anticipating Lagrange, a Jupiter-like body). Newton's laws make explicit that the distance between/among masses affects the accelerations of the masses (or bodies)—i.e. the inverse square distance-force law—and therefore affects their relative falling rates.

3.2 The "Gravity" of Falling Apples…

Newton's theory of gravity is over 300 years old, so old and so *scientifically accepted* that it just *couldn't* be wrong about something as basic as the finding of Galileo that lighter and heavier bodies fall at the same rate, could it?! Well… yes, it could.

Although science is capable of achieving inspiringly great insights, it is also capable of terrifyingly great oversights. Even the greatest of scientists, such as Sir Isaac Newton, have inexplicably made such oversights in their chosen fields of scientific endeavor. In particular, as we are most concerned with here, Isaac (later Sir Isaac) failed to question the famous finding of Galileo—partly experimental, partly

gedanken experimental, partly hypothetical-theoretical (extrapolating to the general case from the experimental)—that lighter and heavier bodies fall at precisely the same rate. If he had, Newton instead of Lagrange would undoubtedly have been credited as the theoretical discoverer of the Lagrangian or Trojan points (L4 and L5; see **Figure 4: The Lagrangian Points L1-L5, p. 139**) and Trojan asteroids.

- **DEFINITION: (to) FALL –** by "(to) fall" we mean that, if an extremely accurate measuring instrument could be fixed to the Earth at the base of the Tower of Pisa that would measure the rate at which the bodies that Galileo dropped instantaneously accelerated toward the measuring device and thus toward the Earth, that measured rate at the instant of release would be the rate of "fall" at that instant.

  Of course, in general "falling" thus means, in more formal-abstract terms, the net acceleration of the point centers of mass toward each other, in particular of each of the 2—lighter/heavier—bodies with respect to e.g. the Earth, the planet Jupiter, the Sun.

  It is important to note that the above means that the falling or acceleration is measured in the "reference frame" of one of the masses that is engaged in the falling or acceleration. Newton theorized (as did Einstein) that all masses fall toward each other, so, retroactively, Galileo is stuck with this definition, even if he thought of the Earth as "unmoving" or "unmoved" by the falling object(s). By now almost anyone can perceive Newton's "… oversight", that, in modern terms, the Earth is in actual fact *not* an inertial frame of reference, i.e. it is *not* a Lorentz frame. All that remains is "haggling over the price…"

The hypothesis that lighter and heavier bodies fall at the precisely the same rate—as we already saw hinted at in the definition just given—has a simple counter-example that should have been noticed by Newton's lesser contemporaries as well as—make that *especially*—by Newton himself:

- the (mass of the) apple (in this case a single apple) falls toward the (mass of the) Earth, but…
- the (mass of the) Earth *also* falls toward the (mass of the) apple (Newton's famously great insight into gravity)
- the Earth will fall faster toward a heavier apple than toward a lighter apple
- therefore the Earth and the heavier apple will fall together faster, both Newton-theoretically… *and* actually
- that is, in sum: the heavier apple falls faster than the lighter apple (Earth relative)

✓ NOTE: when a 1 kg body and a 2 kg body are released in *separate* gedanken trials, the Earth's initial instantaneous acceleration toward the heavier body will be twice its initial instantaneous acceleration toward the lighter body, a difference of ~ $1.64 \cdot 10^{-24}$ m/s^2. In other words, the falling rate difference between a 1 kg body and a 2 kg body—at the "surface of the Earth"—is ~ 1.674 parts in 10^{25} of the standard acceleration of 9.8 m/sec^2 (i.e. the ratio of the 1 kg mass difference between the 2 bodies to the mass of the Earth, independent of units). This would be essentially impossible to measure experimentally, for Galileo or for us today, as it is well beyond our

present experimental accuracy for such things (estimated to be about 1 part in 10^{11}, at least as of the middle of the first decade of the 21st Century).

- ✓ NOTE: this theoretical falling rate difference should also have been noticed and thoroughly analyzed publicly by Einstein—*and especially Eddington*—since it at first seems to contradict relativity theory, which requires that lighter and heavier test particles "accelerate" at the same rate, as they do with Newton but only "relative" to Newton's absolute space-time (frame of reference), which theoretically cannot exist in relativity. This gets sorted out, more or less in relativity's favor, since Earth is not actually a Lorentz frame, which latter is more or less mathematically-logically equivalent to Newton's absolute space-time. By the way, we can note that an "inertial frame of reference" is a purely gedanken concept, unless there can actually exist ("metaphysical") frames of reference that are not pinned to bits of matter-energy which "per force" are constantly accelerating this way and that under the influence of a "superfluity" of "forces".)

It should also have been noticed that when released *simultaneously* (see **Figure 1**, **p. 136**):

- the 2 apples fall toward the Earth,
  but…
- the 2 apples also fall toward each other,
  and at asymmetrically different rates because their masses are different
- since they are equidistant from the center of the Earth, they form an isosceles triangle with the Earth,
  and…
- the falling rates of each apple toward the other are functions of their angular separation (with the Earth as the vertex) and their common distance from the center of mass of the Earth,
  and…
- because neither of the *equal* angles of an isosceles triangle can be a right angle (i.e. both are <°90°),
  we have that…
- the falling rates of each apple toward the other have *different* non-zero components in the direction of the center of the Earth,
  and also…
- theoretically (and actually) the Earth falls ever so slightly faster toward the heavier apple,
  so we…
- add them all up and…
- the 2 bodies fall to Earth at *different* rates *unless*…

Yes, there is an exception, a fascinating one:

- ... *unless* (as we will show later) the 2 apples form with the Earth that special variant of an isosceles triangle, an *equilateral triangle*.

  In this case the 2 apples will fall to Earth at *precisely the same rate*—Newton-theoretically, in our highly abstract gedanken experiment.

  (Even Lagrange himself "… oversighted" that his Trojan points—now more often called Lagrangian points, L4 and L5—have this property

of same falling rates, and they are the only points that do (gedanken Newton-theoretically), and then only when precisely 3 bodies are involved. See **Figure 4: The Lagrangian Points L1-L5, p. 139**.)

- ✓ This falling rate difference can be quite small, to be sure. The acceleration due to gravity at the surface of the Earth is roughly 9.8 m/sec^2, so the theoretical falling rate difference of a 1 and a 2 kilogram mass held 1 meter apart at the surface of the Earth is only about $5.23 \cdot 10^{-18}$ m/sec^2 (very roughly 5 parts in 10^{19}). The accepted "average" acceleration due to gravity at sea level—more or less over the whole surface of the Earth—is slightly greater, by convention 9.80665 m/sec^2; the average value at the poles is considered to be approximately 9.832 m/sec^2, and the average at the equator is considered to be approximately 9.789 m/sec^2. This all gets a wee bit more complicated by the equatorial bulge—the equator is roughly 22 km further from the center of the Earth than the poles are, and by the fact that the effect due to the centrifugal/centripetal force due to the rotation of the Earth at the equator is roughly of the same order of magnitude as the equatorial-pole gravitational difference. We will here go with 9.8 m/sec^2.

Although it is (in most "usual" cases, and generalizing to non-isosceles triangles) very small, the falling rate difference of the 2 bodies has a real chance to make itself noticed in the prolonged fall of bodies in an astronomical orbit. This generally non-zero falling rate difference is in reality essential to the underlying dynamics of Lagrange's Trojan points, L4 and L5 (± 60° angular/orbital separation), and their associated "tadpole" and "horseshoe" orbits. (See **Figure 4: The Lagrangian Points L1-L5, p. 139**.)

In fact, the falling rate difference of lighter and heavier bodies is *far more visibly noticeable astronomically* than the advance in the perihelion of the orbit of Mercury, often described as "infinitesimal" by physicists, since the Trojan asteroids are highly astronomically observable. They can be studied successfully in far less than the century it has taken to get our current estimate of the advance of the perihelion of the orbit of Mercury. In actual fact, many such asteroids orbit a single Trojan point in "tadpole" orbits, and some even orbit *both* Trojan points in "horseshoe" orbits. (See **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**.) The most numerous found so far are those asteroids that inhabit the tadpole orbits around Jupiter's L4 and L5, but others have been found associated with Jupiter's moons, and even Mars and Earth. Although a Trojan asteroid might take hundreds of years to complete its tadpole orbit of e.g. a Jupiter Trojan point, once such an asteroid is detected, it takes far less than a century to determine that orbit. (See **Figure 4: The Lagrangian Points L1-L5, p. 139**.)

By comparison: the observed advance beyond that predicted by Newtonian theory in the perihelion of the orbit of Mercury, that has been studied and approximated over the last 100+ years, is ~ 40 arc-sec/century. If we look at the total angle swept by Mercury in a century, this advance will be on the order of approximately 1 part in 10^8 of that total. But it takes more or less a century to observe carefully enough to approximate that advance accurately (hopefully). Changes and advances in the technology of astronomical observations in the last hundred years have been tremendous, and accumulated errors and inconsistencies, especially from and with early observations, are very difficult to gauge—e.g. has the rate of advance changed with time, like continents drifting apart at different rates in different epochs? *But*… the

perihelion advance is considered to be very close to the value predicted by Einstein, and is taken as support for relativity over Newtonian theory. So, although *small*, it is considered *very important scientifically*.

3.3 "Infinitesimals" and Levels of Approximation

For review: it is a standard practice/technique of calculational convenience in physics (et al) to approximate some quantities, such as relatively very small masses, as "infinitesimals". E.g. an "infinitesimal mass" is one which has "effectively zero" gravitational effect on the other masses—in a technical but strict violation of Newton's Law of Gravity. So an infinitesimal mass would be "small enough" to *not* attract any other masses, but would be "large enough" to *be* attracted by other *non*-infinitesimal masses.

- ✓ This same kind of thing is also done for e.g. "infinitesimal" electric charges, large enough to be affected by the electric field—and thus to gedanken test its strength—but so small that they do not add another electric field and/or gedanken distort the one being tested.

Using infinitesimals can make sub-classes of some problems simple enough to be solved more conveniently, or even "solved" at all. I.e. it can make mathematical analysis possible, or much easier—but it can also make for a potentially much poorer approximation. It can make some calculations much easier and/or faster since potentially very many computations need not be performed. In fact this is how Galileo's experiment is usually implicitly analyzed. The 2 bodies that we gedanken-drop from the Tower of Pisa have only a very small mass compared to the Earth, and they are usually (implicitly) considered to be infinitesimal masses. Since "they are too small to affect" the accelerations of other masses (each other or the Earth), we only get the accelerations of the 2 masses due to the gravity of the Earth that **Eq. 1c** describes (see below, **Section 3.4, Some Basic Equations and Some Simple Equations, p. 44**), where m_2 in **Eq. 1c** would be the mass of the Earth. (Note that we have abstracted out all other masses, which is an unrealistic but very conveniently simplifying assumption. What we hope is that whatever we come up with will survive the inescapable *reintroduction* (by Mother Nature) of everything we have abstracted out, at least to a "good approximation" in at least some "useful situations".)

But… (and this needs to be emphasized):

- ➢ When we say that *both* of the 2 falling bodies are "infinitesimal", we are implicitly assuming that their mass *difference* can be *no greater* than "infinitesimal"; but when we say that 1 of the 2 falling bodies is actually *heavier* than the other, we are "explicitly assuming" that their *mass difference* is *not* "infinitesimal" (i.e. that it is "effectively *not* zero"); i.e. we are assuming rather that this mass *difference* is *infinitely greater* than the assumedly "infinitesimal" masses themselves. "Reductio ad absurdum" anyone?!

  These assumptions are inconsistent, and worse, they prejudice the result so much that we altogether miss an extremely simple approach to Trojan-Lagrangian points and asteroids.

If we have 3 bodies, we can readily discern at least 4 levels of gravitational abstraction and associated approximation:

0) none of the bodies gravitationally affects any of the others

1) 1 of the bodies gravitationally affects the others, but is not gravitationally affected by them
2) 2 of the bodies gravitationally affect each other and the 3rd, but are not gravitationally affected by the 3rd
3) all 3 bodies affect each other gravitationally, as per Newton's laws

We have further:

0) The 0th level of abstraction and associated approximation is not without its important uses since it is actually the kind used in e.g. thermodynamic models of ideal gas kinetics (usually with so many more bodies that only statistical methods are used; this, by the way, has historically led to other… oversights).
1) The 1st level of abstraction-approximation is the level at which Galileo is still "scientifically correct"; it is the level which Kepler implicitly assumed when he had the (center of the) Sun, rather than the common center of mass, at the focus of the elliptical orbits of the planets (well, one planet at a time); it yields a good engineering approximation for non-orbital falls (therefore of short duration) of lighter and heavier bodies.
2) The 2nd level of abstraction-approximation is the level associated with Lagrange's analysis of Trojan points; it does not strictly hold here, though, since it also required (for "stability") that 1 of the 2 non-infinitesimal bodies be of an "intermediate infinitesimality" ($m_2 < \sim 0.04\ m_1$).
3) The 3rd level is the one we will examine here, *and* we will make no assumptions about the relative sizes of the masses. However, neither we will deeply analyze the "stability" of the "equilibrium" (i.e. when "small enough" perturbing influences are acting throughout the not well-defined tadpole or horseshoe regions).

At the 2nd and 3rd levels of abstraction-approximation where 2 or all 3 bodies respectively are considered non-infinitesimal, lighter and heavier bodies exhibit a *mostly* non-zero falling rate difference.

- Here it is also important to note that we are already accepting that the distance from the Earth or Earth-like body, the distance from which we will release the lighter and heavier bodies, does affect the falling rates of those bodies—already a theoretically important deviation from theoretically precisely equal falling rates.

As we noted earlier, the falling rate difference is a function of their angle of separation. (Actually, using the angle of separation is not theoretically necessary, but in our case a *quite* convenient computational choice.) Thus, we abstract to a situation where the lighter and heavier bodies are point masses equidistant from the point mass of the Earth, and thus that the 3 point masses form an isosceles triangle (2 equal sides and 2 equal angles, with the possibility of a third of each).

When the falling rate difference (*due to the mass difference*) is studied, one finds that:

- it is proportional to the mass difference

- it is inversely proportional to the square of the common distance of the 2 masses from the point center of mass of e.g. the Earth
- it is a function of the angular separation of the 2 bodies (with e.g. Earth as the vertex)
- it changes sign between 0 and ± 180 degrees, and, in fact,…
- it zeroes at precisely ± 60 degrees

Now things start to get interesting because ± 60 degrees are the angles associated with the Trojan points (L4 and L5) predicted by Lagrange, using his extremely arcane perturbation theory. The falling rate difference shows itself during the prolonged fall of orbiting bodies, most visibly in the dynamics of those mysterious and fascinating bodies today known as Trojan asteroids. (See **Figure 4: The Lagrangian Points L1-L5, p. 139**.)

Lagrange's result concerning stability, however, assumes that there are 2 very large masses, with 1 very much smaller than the other, $m_2 < \sim 0.04\, m_1$. He analyzed the planet Jupiter—our largest planet, about 0.1% the mass of the Sun—in orbit around the Sun. And it was a long time before people started thinking about Trojan asteroids associated with other planets or moons, because Jupiter seemed so massively unique.

- ✓ See the *extremely simple* argument for the stability of a Trojan system *of arbitrary masses* offered in **Section 3.12, Stability?!, p. 62**. It may turn out, however, that stability does need the big guns of something like Lagrange's perturbation theory and mass restrictions to get everything right, although the stability calculation would probably also succumb to a very detailed algebra-trig approach like that *started* here. Remember, the case presented here is a *static 2*-dimensional case, but as soon as everything starts spinning and the bodies—*all 3!*—experience perturbations (either somewhat realistic, such as the sudden appearance of another mass or a firing rocket motor attached to the mass at that point, or non/less-realistic such as having one of the masses do a quantum jump to another position, or having its velocity make a quantum jump to another velocity), the problem becomes a *dynamic 3*-dimensional case, with many more and much messier details to wade through and get right.

Lagrange studied what would happen to *infinitesimal* bodies (see the list of levels of abstraction, above) that found themselves near certain points in relation to the 2 other very much larger masses. (Even large asteroids were/are considered infinitesimal—in comparison to Jupiter—for purposes of Trojan point analysis.) In particular, Lagrange studied what happens when the 3 bodies, subject to the above mentioned restriction on relative mass (the "intermediate infinitesimality" of the second largest mass), form an equilateral triangle and concludes that such a point in relation to the others is a stable one for an infinitesimal body to occupy. I.e. the infinitesimal body will remain at or near the Trojan point even if it is (merely) modestly perturbed by other forces. This means that asteroids will orbit (in a fascinating "tadpole" orbit; see **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**; this is a more detailed picture than Lagrange was able to compute-predict), or *possibly* equilibrate at, the 2 equilateral triangle points (the "Trojan points"; see **Figure 4: The Lagrangian Points L1-L5, p. 139**) in the orbit of Jupiter, the L4 point leading Jupiter by 60 degrees, the L5 point trailing it by 60 degrees, even when such asteroids were "perturbed" (within "limits"). It can take hundreds of years for a Trojan

asteroid to complete such an orbit, even if it stays in its tadpole, not orbiting *both* L4 and L5 in a horseshoe orbit.

But it is actually very simple to demonstrate the result that, *if unperturbed*, any 3 *arbitrary* masses can remain in an equilateral triangle in an equilibrium orbit around their common center of mass (sometimes called their "barycenter"). We can even have an equilateral triangle that is expanding and contracting "homographically" (maintaining geometric similarity, shape but not necessarily size, which here can mean cyclical expansion and contraction). In fact, it is easy to see that if the proper initial velocities are given, i.e. any velocities greater than their escape velocities, the equilateral triangle can be made to expand to infinity, either with no rotation or with an ever slower rotation around the center of mass.

Although we are here still missing the crucial aspect of stability, this result still has fascinating implications for both professional and amateur astronomy in the 21st Century. We should remember that stability will always be relative to a combination of the magnitude and direction of the perturbing forces, the depth of the potential energy well involved, and the time it takes for the perturbing forces to have effect; i.e. if the perturbing force vector is great enough in the right direction, *no* perturbed body orbiting in an associated potential energy well will be "stable". Even with a small potential energy well, but a correspondingly smaller perturbing force, any body and associated orbit can potentially remain "stable".

We have to remember that Lagrange, too, missed the now almost intuitively obvious scientific fact that lighter and heavier bodies fall at different rates *and* that this gives rise to the Trojan points, so… perhaps he also missed other interesting things in his analysis.

3.4 Some Basic Equations and Some Simple Equations

Lagrange's perturbation theory is notoriously complex and difficult, even for professional physicists, astronomers and mathematicians. Also notoriously complex and difficult for many is the mathematics of (partial) differential equations, which is usually used to prove one of the results that will be demonstrated here, i.e. that 3 arbitrary masses equidistant from each other will theoretically remain equidistant from each other if given the proper initial positions and velocities—given that they remain free from perturbations, which never happens in the real world. Unfortunately, the differential equation approach also misses the essential falling rate difference.

Here, however, this unperturbed Trojan point result will be demonstrated using only Newton's laws, high school algebra and trigonometry, and of course the preliminary result (already seen above) that lighter and heavier bodies fall at different rates except at the Trojan points. The usual differential equation approach is not necessary. Not even basic calculus is truly *needed*, not Newton's, not even Leibniz's. It is hoped that this will make the study of the physics of Trojan points and Trojan asteroids more accessible, even to high school and other beginning physics and astronomy students. It is also hoped that the derivation of these results from *critically* questioning the accepted "scientific law" of equal falling rates will be an inspiration to *all* students, teachers, and practitioners, of any level of experience or sophistication, of any subject, especially science. "Science… when wrong, to be put right."

The only mathematics and equations we need for a fascinating reexamination of the falling bodies of Galileo, and a first examination of Newton's oversight, are quite simple and well known today, even by high school standards. They are about as simple as mathematics and equations can be and still allow one to do useful science. Any first year undergraduate physics or astronomy major (or even a high school honors physics

student), even in a more lenient American university or college, would be expected to be able to readily handle such equations… or change majors.

First we have:

Equations 1: Newton's most well known laws of mechanics and gravity (see **Eqs. 1a** and **1b**, below)

Assumptions 1:

Newtonian gravity, no relativistic effects, infinite speed of gravity (i.e. we ignore that the speed of gravity is finite)

abstract out all other forces: gravitational anomalies, Coriolis, buoyancy, wind, viscosity, electromagnetic, thermodynamic, etc.

2 point masses (constant) representing the lighter and heavier bodies

1 point mass (constant) representing the Earth (constant)

the 2 point masses will be released at equal distances from the point mass representing the Earth

Variables 1:

m_i — *m*asses, where a number indicates a generic mass, a letter a particular mass (Earth, Lighter mass, Heavier mass)

F — *F*orce

a_i — *a*ccelerations, where a number indicates the acceleration of a generic mass, a letter the acceleration of a particular mass (Earth, Lighter mass, Heavier mass)

G — Newton's *G*ravitational constant

r — the *r*adial (linear) distance (between the masses)

First we have Newton's basic equation of motion:

Eq. 1a: $F = ma$

Then we have Newton's equation for his law of gravity:

Eq. 1b: $F = G\frac{m_1 \cdot m_2}{r^2}$

where the force on each mass is directed toward the other mass (and thus directed toward the center of mass; this is very important because when there are more than 2 masses, the forces will be directed toward the common center of *all* the masses only if *all* the masses are in very special arrangements). Since this last equation gives us the force on each mass represented, combining them we get:

$$F = m_1 a_1 = m_2 a_2 = G\frac{m_1 \cdot m_2}{r^2}$$

and thus the acceleration of each mass is:

Eq. 1c: $a_1 = G\frac{m_2}{r^2}$ and $a_2 = G\frac{m_1}{r^2}$

Note that, by **Eq. 1c**, the (*initial*, *instantaneous*, and, most importantly, *absolute*) acceleration of each body in the gravitational field does not depend on its own mass, but only on the mass of the other body (or bodies), their mutual distance(s), and of course the gravitational "constant".

- ✓ It is essential regarding the relevance to relativity of this acceleration, at least of its equation, to note that it is purely in terms of the *absolute* Newtonian space-time frame of reference, which theoretically cannot exist in relativity. And at least *some* astronomers and cosmologists keep wondering from time to time whether the gravitational constant is actually constant; some are now even including it as a parameter in calculations relating to "dark matter".

It is this last equation (**Eq. 1c**), which corresponds to abstraction-approximation level 1 (as we saw above in **Section 3.3, "Infinitesimals" and Levels of Approximation, p. 41**), that taken by itself has lead many people to believe that Galileo was scientifically correct, and that lighter and heavier bodies will fall at precisely the same rate (with the implicit assumption that both those masses are "infinitesimals"; see comment in **Section 3.3, p. 41**). But in reality it is only one of many force components that might act on each body, including on the Earth. Since falling is e.g. Earth relative, we must also take *its* acceleration into account in a complete analysis. We start with the generic equation:

Eq. 1d: $a_1 + a_2 = G\frac{m_2}{r^2} + G\frac{m_1}{r^2} = (m_2 + m_1)\frac{G}{r^2}$

Then, taking the acceleration of the Earth into account, and taking the difference of the falling rates of the Heavier and Lighter masses:

Eq. 1e: $(a_E + a_H) - (a_E + a_L) = a_H - a_L = (m_H - m_L)\frac{G}{r^2}$

Then, finding the ratio of the difference of the falling rates to the standard acceleration (of an "infinitesimal" mass) due to gravity of Earth at its surface ($a_E \equiv g$):

Eq. 1f: $(a_H - a_L)/a_E = \frac{(m_H - m_L)}{m_E}$

Here we find ourselves assuming that $0 < m_L < m_H << m_E$ although strictly we don't need to do so.

Eq. 1d shows the falling rate or relative acceleration of 2 bodies in the reference frame of either of the bodies, as opposed to the absolute acceleration of (either) 1 of the bodies in the absolute Newtonian frame of reference. **Eq. 1e** shows the difference in falling rates, Earth relative, of 2 masses (released separately), one heavier, one Lighter. **Eq. 1f** shows the ratio of the falling rate difference to the absolute acceleration due to Earth's gravity (assumed to make up the vast majority of the relative falling rate). Note that it doesn't depend on either the distance *r* or on the gravitational constant *G*, just on the relative masses. Since the mass of the Earth is

$\sim 5.975 \cdot 10^{24}$ kg, the falling rate difference between a 1 kg mass and a 2 kg mass is ~ 1.67 parts in 10^{25}. Newton should never have overlooked this. Einstein should never have overlooked this, and Eddington… well, it goes without saying.

We will continue here the usual policy and abstract out viscosity, buoyancy, gravitational "anomalies" (other masses and/or non-uniform density distributions of mass), the speed of gravity, etc. We will consider only the Newtonian-gravitational forces of the 3 bodies (taken as point masses) at their centers of mass. But, we still get *2* forces acting on each body in this, our Galileo's Tower of Pisa gedanken experiment: the gravitational forces from the other 2 of the 3 falling bodies. This corresponds to our level 3 of abstraction-approximation (as we saw above in **Section 3.3, "Infinitesimals" and Levels of Approximation, p. 41**).

Most people ignore the fact that the Earth falls, even though Newton's theory says it does. And in our case it falls/accelerates due to gravity at a non-zero rate toward the 2 bodies. It falls slightly faster toward the heavier one. And even more importantly, the 2 bodies whose falling rates we are comparing fall toward each other. Although we could "get away with it" and ignore these terms as physicists do in the simpler levels of abstraction-approximation, here we will choose not to. In fact we need to take *all* the forces (of our *abstract* case) into account to find the easy, low road to the Trojan points—crediting Lagrange with taking the difficult, high road—the low road that Newton, of all people, should *never* have missed… but *did*.

3.5 Equations for a *Simple* 3-Body Problem

Referring to **Figure 1**, **p. 136**, of the 6 acceleration vector components there are 8 sub-components, **Eqs. 2a** through **2h**, that are needed to compute the net accelerations of the bodies toward each other at the instant of release—we are not going to trace their trajectories, here—of both the lighter body and the Earth toward each other, and the heavier body and the Earth toward each other. Some of these will be a function of the angle between the 2 bodies with the Earth point center of mass as the vertex. (**Eqs. 2a** and **2b** were already discussed above as **Eq. 1c**.)

Equations 2: The 8 sub-components of the accelerations of each of the lighter and heavier bodies and the Earth toward each other, and their sums and differences.

Assumptions 2: same as Assumptions 1, except:

3 point masses, arranged in an isosceles triangle (masses constant)

2 dimensions will be sufficient for our purposes, even with 3 point masses (the 3 points determine a plane, they will all start with zero velocity, and there will be no forces or movements directed out of the plane)

Variables 2:

m_L — the *m*ass of the *L*ighter body

m_H — the *m*ass of the *H*eavier body

m_E — the *m*ass of the *E*arth ($\sim 5.975 \cdot 10^{24}$ kg)

r — the distance from the Earth to either body (same distance in this case)

θ — the angle between the lines between each body and the Earth (as the vertex)

Eq. 2a: $a_{LEE} = G \cdot \frac{m_E}{r^2}$ the acceleration of m_L toward m_E due to m_E

Eq. 2b: $a_{HEE} = G \cdot \frac{m_E}{r^2}$ the acceleration of m_H toward m_E due to m_E

Eqs. 2a and **2b** are both the standard "acceleration due to gravity" (if *r* is the radius of the Earth) which are the same for both the lighter and heavier bodies. Note that neither acceleration is a function of the angle θ. Near Earth (i.e. in most sub-orbital situations) they are adequate as an engineering abstraction-approximation, but we are interested here in a simple mathematical description of the subtle differences between the standard abstraction-approximation and actual reality that will lead to the Trojan points.

The 6 following equations are the accelerations due to gravity that are standardly ignored as "infinitesimal" or zero:

Eq. 2c: $a_{ELL} = G \cdot \frac{m_L}{r^2}$ the acceleration of m_E toward m_L due to m_L

Eq. 2d: $a_{EHH} = G \cdot \frac{m_H}{r^2}$ the acceleration of m_E toward m_H due to m_H

We see that the Earth falls faster toward the heavier body. Again, neither acceleration is a function of the angle θ. This changes for the next 4 accelerations.

Eq. 2e: $a_{ELH} = G \cdot \frac{m_H}{r^2} \cdot \cos(\theta)$ the acceleration of m_E toward m_L due to m_H

Eq. 2f: $a_{EHL} = G \cdot \frac{m_L}{r^2} \cdot \cos(\theta)$ the acceleration of m_E toward m_H due to m_L

This is a good place to remind ourselves that there is no necessity that the mass designated as "Earth" must be the greatest mass, so this result will depend neither on relative mass nor on the "infinitesimality" of any of the 3 masses. If it is by far the greater mass of the 3, then the terms from **Eqs. 2e** and **2f** will of course be much smaller than those from **Eqs. 2g** and **2h**, *unless* the angle between them is *very* small. When that happens they accelerate very rapidly toward each other and this has a *large* component in the direction of the Earth, as we will see next.

Eq. 2g: $$a_{LEH} = G\frac{m_H}{\left(2r\sin\left(\left|\theta/2\right|\right)\right)^2} \cdot \sin\left(\left|\theta/2\right|\right) = G\frac{m_H}{r^2} \cdot \frac{1}{4\sin\left(\left|\theta/2\right|\right)}$$

the acceleration of m_L toward m_E due to m_H

Eq. 2h: $$a_{HEL} = G\frac{m_L}{\left(2r\sin\left(\left|\theta/2\right|\right)\right)^2}\cdot\sin\left(\left|\theta/2\right|\right) = G\frac{m_L}{r^2}\cdot\frac{1}{4\sin\left(\left|\theta/2\right|\right)}$$

the acceleration of m_H toward m_E due to m_L

- ✓ NOTE: there is a subtlety in the system of angles in this problem such that, if we switch the positions of the 2 bodies, the angle that affects these last 2 equations (not actually the angle in **Figure 1**, **p. 136**, labeled $\theta/2$) changes sign. The results are in fact symmetrical, and it is easiest to take the absolute value to deal with the seeming asymmetry.

When we combine all the terms to get the total accelerations of each body and the Earth toward each other, we get:

Eq. 2i: $$a_{LT} = a_{LEE} + a_{ELL} + a_{ELH} + a_{LEH}$$
the total acceleration of m_L and m_E toward each other

Eq. 2j: $$a_{HT} = a_{HEE} + a_{EHH} + a_{EHL} + a_{HEL}$$
the total acceleration of m_H and m_E toward each other

Combining these to get the difference of falling rates of the heavier and lighter bodies we get:

$$a_{H-L} = a_{HT} - a_{LT} = a_{HEE} - a_{LEE} + a_{EHH} - a_{ELL} + a_{EHL} - a_{ELH} + a_{HEL} - a_{LEH}$$

- ✓ **NOTE:** the 2 terms representing the usual acceleration due to the mass-gravity of Earth cancel.
- ✓ **NOTE WELL:** these 2 canceled terms are the *only* terms that depend on m_E, the Earth's mass.
- ✓ **NOTE also:** when performing a calculation on a computer it is best to leave these terms out to help minimize floating point errors (and inaccuracy).

Proceeding we get:

$$a_{H-L} = a_{EHH} - a_{ELL} + a_{EHL} - a_{ELH} + a_{HEL} - a_{LEH}$$

and expanding which we get:

$$a_{H-L} = G\cdot\frac{m_H}{r^2} - G\cdot\frac{m_L}{r^2}$$

$$-G\cdot\frac{m_H}{r^2}\cdot\cos(\theta) + G\cdot\frac{m_L}{r^2}\cdot\cos(\theta)$$

$$-G\cdot\frac{m_H}{r^2}\cdot\frac{1}{4\sin\left(\left|\theta/2\right|\right)} + G\cdot\frac{m_L}{r^2}\cdot\frac{1}{4\sin\left(\left|\theta/2\right|\right)}$$

Eq. 2k: $$a_{H-L} = \frac{G}{r^2} \cdot \left(m_H - m_L\right) \cdot \left(1 - \cos(\theta) - \frac{1}{4\sin\left(|\theta/2|\right)}\right)$$

which is the simplified expression for the difference of the falling rates of the heavier and lighter bodies when released simultaneously with an angle of separation of θ. (See **Figure 2**, **p. 137**, for a plot of the falling rate difference as a function of the angle between 2 sample unequal masses.)

- ✓ **NOTE:** there are 2 factors that can be zero:
  1) if the masses are equal then their difference is zero, *and* the falling rate difference *due* to their mass difference is zero, as it ought to be;
  2) if the trigonometric factor is zero, the falling rate difference will be zero. It is easy to calculate that the trig factor will zero if and only if θ is ± 60° (i.e. $\theta = \pm \pi/3$ radians).

Eq. 2m: $$f(\theta) = 1 - \cos(\theta) - \frac{1}{4\sin\left(|\theta/2|\right)}$$, the trigonometric factor

- ✓ and we note that:

$$\begin{aligned} f(\pm 60°) &= 1 - \cos(\pm 60°) - 1/(4\sin(|\pm 60°/2|)) \\ &= 1 - 1/2 - 1/(4 \cdot 1/2) \\ &= 1 - 1/2 - 1/2 \\ &= 0 \end{aligned}$$

It is a simple exercise in trigonometry to show that ± 60° are the only 2 roots of $f(\theta)$. These are the angles—and the only angles—which place all 3 bodies at the vertices of an equilateral triangle with regard to one another, i.e. 1 of them is in either Lagrangian point L4 or L5 relative to the other 2. Refer to **Figure 2: The Difference in Falling Rates as a Function of Angular Separation, p. 137**.

So, we have shown that:

- **Galileo was in fact *scientifically incorrect*** about lighter and heavier bodies falling at (precisely) the same rate.
  If we consider just 3 point masses and Newtonian gravity:
  - the *heavier* body and the Earth will fall together faster if they are further apart than 60° (with the center of mass of the Earth as the vertex)
  - only at *precisely* 60°—i.e. the only place where Galileo is scientifically correct—will they fall at *precisely* the same rate (approximately)
  - and paradoxically, when they are closer together than 60° the *lighter* body and the Earth will fall together faster

Humorous remarks are possible about Aristotle being correct 2/3 of the time ($60° < \theta < 300°$), Galileo being correct only on a set of "measure zero" ($+60° \le \theta \le +60°$ and $-60° \le \theta \le -60°$), and, ironically, *neither* being correct 1/3 of the time ($-60° < \theta < +60°$), which would have pertained if Galileo had actually

performed his apocryphal Tower of Pisa experiment of releasing both masses simultaneously, but we will not indulge… at this time.

We will quickly use **Eq. 2k** to get the falling rate difference for our Tower of Pisa case, with Galileo holding a (dense) 1 kg mass in one hand and a (dense) 2 kg mass in the other, 1 meter apart at the top of the tower.

Equations 3: The falling rate difference for the Tower of Pisa gedanken experiment.

Assumptions 3: same as Assumptions 2

Variables and Constants 3:

m_L — m_L = 1 kg, the *m*ass of the *L*ighter body

m_H — m_H = 2 kg, the *m*ass of the *H*eavier body

θ — the angle between the lines from each body to the Earth, i.e. with Earth as the vertex

r — r = 1 m (meter), the distance between the lighter and heavier bodies

G — $G \cong 6.67259 \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{sec}^2}$, the universal gravitational constant

R_E — $R_E \cong 6.378 \cdot 10^6$ m, the Radius of the Earth

g — $g \cong 9.8 \frac{\mathrm{m}}{\mathrm{sec}^2}$, the acceleration due to gravity at the top of the Tower of Pisa, more or less the same as at the surface of the Earth

Eq. 3a: $$a_{H-L} = \frac{G}{R_E^2} \cdot \left(m_H - m_L\right) \cdot \left(1 - \cos(\theta) - \frac{1}{4 \sin\left(\left|\theta/2\right|\right)}\right)$$

(from **Eq. 2k**)

Eq. 3b: $$\frac{G}{R_E^2} \cong \frac{6.67259 \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{sec}^2}}{\left(6.378 \cdot 10^6 \text{ m}\right)^2} \cong 0.164 \cdot 10^{-23} \frac{\mathrm{m}}{\mathrm{kg} \cdot \mathrm{sec}^2}$$

Eq. 3c: $$m_H - m_L = 1 \text{ kg}$$

Eq. 3d: $$\cos(\theta) \cong 1 - \theta \cong 1 - 2 \cdot \sin\left(\left|\theta/2\right|\right) \cong 1 - 1.568 \cdot 10^{-7} \cong 1, \text{ in our case}$$

Eq. 3e: $$\sin\left(\left|\theta/2\right|\right) = \frac{r}{R_E} \cong \frac{0.5 \text{ m}}{6.378 \cdot 10^6 \text{ m}} \cong 7.84 \cdot 10^{-8}$$

Eq. 3f: $$a_{H-L} \cong \left(0.164 \cdot 10^{-23}\right) \cdot (1) \cdot \left(1 - 1 - \frac{1}{4 \cdot 7.84 \cdot 10^{-8}}\right) \frac{\mathrm{m} \cdot \mathrm{kg}}{\mathrm{kg} \cdot \mathrm{sec}^2}$$

$$\cong -0.164 \cdot 10^{-23} \cdot 3.189 \cdot 10^{6} \frac{\mathrm{m}}{\mathrm{sec}^2} \cong -5.23 \cdot 10^{-18} \frac{\mathrm{m}}{\mathrm{sec}^2}$$

Eq. 3g: $$\frac{a_{H-L}}{g} \cong \frac{-5.23 \cdot 10^{-18} \frac{\mathrm{m}}{\mathrm{sec}^2}}{9.8 \frac{\mathrm{m}}{\mathrm{sec}^2}} \cong -5.337 \cdot 10^{-19}$$

which last is the ratio of the falling rate difference of a 1 kg mass and a 2 kg mass held 1 meter apart and released simultaneously at the Earth's surface (the top of the Tower of Pisa) to $g \cong 9.8\,\mathrm{m/sec^2}$, the more or less standard acceleration due to gravity at the Earth's surface.

3.6 A Quick Look at the Separate Release Case… and Einstein's "Relativity"

Before moving on, we will take another quick look at the separate release case, by looking at **Eq. 2k**. If we ignore the angle that pertains to simultaneous release, this equation gives us the gives us a difference in the accelerations of the Earth toward the lighter and heavier bodies, which difference is trivially non-zero.

- The ratio of the acceleration difference to the acceleration due to Earth's gravity is trivially the ratio of the mass difference of the lighter and heavier bodies to the mass of the Earth. If we take the mass of the Earth as $5.975 \cdot 10^{24}$ kg, a 1 kg mass difference will give rise to an acceleration difference that is about 1.674 parts in 10^{25}, independent of the *common* distance of the bodies from the Earth.

Only **Eqs. 2a** and **2b** make it seem that the lighter and heavier bodies experience the same acceleration. But they also make clear the fact that this is only relative to Newton's absolute space-time frame of reference, not what one can call a kosher "relativistic" frame of reference.

This last brings up an interesting gedanken possibility. Although it is pretty much purely theoretical, we can perform a gedanken experiment such that if one can find a frame of reference in which the lighter and heavier bodies (of precisely 3 bodies) accelerate at the same rate (not counting the case of precisely 3 bodies in an equilateral triangle), then that frame of reference must be an absolute Newtonian-style frame of reference. There is no way in the real world to get an all-gravitational-conditions-equal experimental framework for the separate releases of 2 or more (unequal) masses, but gedanken experiments have no such limitations.

3.7 Homographic Property—Maintaining Shape (Geometric Similarity)

So, we have just seen that it is simply derivable that if all 3 bodies form an equilateral triangle, then the Earth and the lighter body will fall together at the same rate as the Earth and the heavier body, or rather that they will start to when all 3 are released simultaneously—we don't even need calculus to figure that out. We will see shortly that they will continue to do so.

Since we are trying to make a point here of how easily this oversight could have been avoided—or, more optimistically, how easily this discovery could have been made—even in Newton's day, we will note that the relations among acceleration, velocity and distance were sufficiently well known long before Newton's—and Leibniz's—development of calculus. They were more than simple enough as far as Newton's scientific contemporaries are concerned, even if calculus was eventually needed to provide a satisfying mathematical foundation for them.

Now we can note again that the equations we have just looked at do not actually distinguish the 3 bodies in the equilateral triangle—except by mass—and their falling rates are the same, *independent* of mass. It takes only a quick mental switch to see that, starting from an equilateral triangle, the "lighter body" and the "heavier body" will fall together at the same rate as the "Earth" and either body. So with almost no extra calculational effort we now have the situation that, at least if they start at rest, all 3 bodies will fall together toward the common center of mass maintaining the shape of the equilateral triangle that they start in, where each of them is in either Lagrangian point L4 or L5 (Lagrange's Trojan points; see **Figure 4: The Lagrangian Points L1-L5, p. 139**) with regard to the other 2 bodies. Therefore they not only all start to fall together at equal rates—i.e. each pair falling together at the same rate as the other pairs—they continue to do so.

But the question remains whether they will remain in this equilateral triangle configuration under other circumstances.

- Digression: perhaps you are wondering how it is known that they will fall toward their common center of mass. This follows simply from Newton's well-known laws of motion which we have heretofore avoided mentioning. There is no external force acting on the 3 masses, so their center of mass, initially at rest, will stay at rest. We also know that they must all fall toward a common point as the equilateral triangle shrinks to a point. So, by an argument such as Newton or his contemporaries might have offered, this common point must be their common center of mass.
- Further Digression: a more subtle point, and an essential one, is that, when the 3 point masses are in an *equilateral* triangle, then—independent of the masses—each mass, the center of mass of all 3 masses, and the center of mass of the other 2 masses are (all 3) collinear (and in that order)!

The maintenance of the *same shape* of the equilateral triangle as the 3 bodies move through space is the *homographic* property that Lagrange found them to have, even when they were revolving around their common center of mass. The question comes up as to how *simply* and *easily* it can be shown—using only such arguments such as Newton might have used—that all 3 bodies will homographically remain at the vertices of an equilateral triangle, even when revolving, even when revolving with expansion and contraction. Reminder: Lagrange already showed that this does happen, using his perturbation theory, but we are hoping to show here that Newton—or even his lesser contemporaries—could have discovered the Trojan points using only the mathematics of their time, preferably just using algebra and trigonometry, avoiding even Newton's or Leibniz's calculus, not to mention partial differential equations, the calculus of variations and perturbation theory.

Some of you may have already objected that vector arithmetic had not yet been invented in their day, and this is correct in a strict sense. But they were familiar with surveying, architecture, engineering, the trigonometry to do related calculations, and

with "*something*" corresponding to the associated "force vector components", the mathematics of which even then included something more or less equivalent to rudimentary vector arithmetic. Our concept of vectors is mainly relatively *explicit*, and only slightly conceptually superior, so we can use it without violating our intended spirit of simplicity and ease.

3.8 Geometry and the Homographic Property

Here is a good place to remind ourselves of the:

- Necessary and Sufficient Geometric Conditions for Homography

  for the 3 point masses to homographically remain in an equilateral triangle—i.e. maintain geometric similarity, even when changing size and orientation—while revolving around their common center of mass (with no perturbing forces):

  1) the angles formed by the lines joining the common center of mass to each body must remain the same
  2) the ratios of the distances from the common center of mass to each body must remain the same

- ✓ **Reminder:** we have, with reasonable safety for our purposes, assumed that the masses will remain constant over time although this is *not* generally true for astronomical bodies, especially over long periods of time.

3.9 Equations for Homographically Maintaining an Equilateral Triangle Without Expansion and Contraction

We need to look at the equations relating to angular velocity and acceleration. We have the equation for the acceleration of an object moving at velocity v in an orbit of radius r:

Equations 4: relating angular velocity and linear acceleration

Assumptions 4: circular motion

Variables 4:

a — *a*cceleration (radial, toward the center of the circle)

v — *v*elocity (tangential, perpendicular to radius)

r — the *r*adius (of curvature) of the circle

θ — the angular distance

$\dot{\theta}$ — the angular velocity, $\dot{\theta} = d\theta / dt$ (Newtonian = Leibnizian notation)

m_i — *m*asses

Eq. 4a: $a = v^2 / r$

- ✓ **NOTE:** in this last equation, the direction of the acceleration a for a mass circularly revolving around a point is toward the center of the revolution/circle. It is commonly known that, if there are 2 masses, this will also be toward the common center of mass, although we have not yet shown this here.

And we have the equation relating the velocity v in an orbit of radius r with the angular velocity θ:

Eq. 4b: $$v = r \cdot \dot{\theta} = r \frac{d\theta}{dt}$$

- ✓ **NOTE:** Although we could have used a named constant for the angular velocity, we have slipped into both Newton's—$\dot{\theta}$—and Leibniz's—$d\theta / dt$—calculus notation. The rationalization is that the *notation* helps give a nicely clearer picture what is happening, but that we aren't *really using* calculus for the derivation.

And so we have the equation relating the radial acceleration to the radius r with the angular velocity $\dot{\theta} = d\theta / dt$:

Eq. 4c: $$a = v^2 / r = r \cdot \dot{\theta}^2$$

And we have that the above centripetal acceleration a of a mass in a circular orbit is linearly proportional to its distance r from the center around which it revolves (see *rightmost* term of equation), and that otherwise it is a function only of $\dot{\theta}$. Not shown here is the well-known result that the direction of this acceleration is radial and toward the center of revolution, which in our case will be the common center of mass (here only of 2 bodies). This is one of the fundamental equations used to calculate orbital velocities for satellites.

It is shown below—and also could have been easily shown in Newton's time—that in our special circumstances of 3 equidistant bodies, the acceleration due to gravity of each mass is toward their common center of mass. If we can show that this acceleration is proportional to the distance from that common center, then, combining this with the result just obtained (**Eq. 4c**), we will have shown that the 3 masses can revolve around that common center of mass in such a way as to completely balance the accelerations due to gravity for a calculable constant angular velocity $\dot{\theta}$ maintaining the equilateral triangle (ignoring expansion-contraction for the moment).

IMPORTANT: In fact we will know more than that: we will know that since both the acceleration due to gravity and the acceleration due to revolution will only be radial, i.e. only along the lines toward the common center of mass, that it is then *impossible*—barring other "perturbing" forces acting—for the angles among these lines to change. This last is one of the 2 necessary and (together) sufficient conditions for the masses to remain in an equilateral triangle. And since both forces/accelerations (gravitational and rotationally induced, centripetal and centrifugal) are both radial from the common center of mass and proportional to the radial distance r, then as those radial distances change, they will remain in the same proportions. This is the other of the 2 conditions.

If you objected that we have not quite pinned it down, you are partly correct: it is possible to give an initial velocity to each mass such that they will not remain in an equilateral triangle. But the point we are making here is that there does exist a set of initial conditions that will allow them to remain in an equilateral triangle, even when

that triangle is rotating and expanding/contracting. This will be when the initial angular velocities are equal, which means that the initial tangential velocities are proportional to the radial distances (from the common center of mass), and the initial radial velocities are also in the same proportions as the radial accelerations and the radial distances. The radial/tangential velocities and their changes remain proportional to the radial/tangential accelerations which remain proportional to the radial distances. Thus the equilateral triangle formed by the masses will stay an equilateral triangle, of the same size as it rotates if the initial radial velocities are zero (trivially proportional), and expanding and contracting if the non-zero initial radial velocities are proportional to the radial distances. (You may think this needs calculus to pin down, but it follows very simply from the relations among acceleration, velocity and distance, well know long before Newton's calculus.)

We know, of course, that the angular velocity $\dot{\theta}$, not to mention the distance r, will not remain constant if the triangle expands or contracts. This all means that if we can show—and we wish to do it quite simply—that the radial acceleration due to gravity of each mass is proportional to the (radial) distance from the common center of mass, we will have shown the dynamic homographic quality of the equilateral configuration of *any* 3 arbitrary masses, and therefore the existence of *at least* "unstable/neutral equilibrium" at the Trojan points (i.e. with masses not perturbed from the equilateral triangle) and therefore also the *possibly* "*stable"* equilibrium" existence of Trojan planets/asteroids—which possibility should have been noticed by Newton at least, if not his contemporaries. (The equilibria at Lagrangian points L1-3 are considered to be "unstable", i.e. that a body at one of those points will wander off if not actively kept there. See **Figure 4: The Lagrangian Points L1-L5, p. 139**. This deserves to be called into question, if only to understand the situation better using this new, simple analysis, especially for L3 which we will see lies in the horseshoe orbit of Trojans.)

We will not attempt to examine "stable equilibrium (better, equilibria)" at the Trojan points, where the positions, velocities and/or accelerations of the masses are perturbed. And of course, here we have continued to assume that the velocity of gravity is infinite. The more explicit one is about dependency on assumptions (both in general and in particular), the less deadly and more friendly those assumptions can be in the long run!

Referring to **Figure 3: The 3 Masses and 2 of Their Centers of Mass, p. 138**, we will look at the equations that describe the acceleration due to gravity of one of the masses, m_3. We can simplify the equations greatly if we allow $G = 1$ as the gravitational constant and $r = 1$ as the distance among the each pair of the 3 masses. The accelerations a_{3i} of m_3 due to m_i are (scalar):

Equations 5: accelerations of the 3rd of 3 masses, special case of equilateral triangle

Assumptions 5: same as Assumptions 2 and Assumptions 3, and, in addition,

$G = 1$ and $r = 1$ (to simplify the calculations)

Variables 5:

G — the *G*ravitational constant (here = 1)

r — the *r*adial (linear) distance (between the masses; here = 1)

m_i — *m*asses

a_{3i} — *a*ccelerations of mass 3 toward mass i

a_{3ix} and a_{3iy} — the x and y axis direction components of a_{3i}

ϕ — the angle with m_3 as the vertex from the y-axis to the common center of mass

Eq. 5a: $|a_{31}| = G \cdot m_1 / r^2 = m_1$ and $|a_{32}| = m_2$

The components in the x and y directions are:

Eq. 5b: $|a_{31x}| = \sin(30^\circ) \cdot |a_{31}| = \sin(30^\circ) \cdot m_1$ and

$|a_{32x}| = \sin(-30^\circ) \cdot |a_{32}| = -\sin(30^\circ) \cdot m_2$

and

Eq. 5c: $|a_{31y}| = \cos(30^\circ) \cdot m_1$ and

$|a_{32y}| = \cos(-30^\circ) \cdot m_2 = \cos(30^\circ) \cdot m_2$

If we sum the components in the x and y directions we get:

Eq. 5d: $|a_{3x}| = \sin(30^\circ) \cdot m_1 + \sin(-30^\circ) \cdot m_2 = \sin(30^\circ) \cdot (m_1 - m_2)$ and

$|a_{3y}| = \cos(30^\circ) \cdot m_1 + \cos(-30^\circ) \cdot m_2 = \cos(30^\circ) \cdot (m_1 + m_2)$

The absolute value of the acceleration of m_3 is:

$$|a_3| = \sqrt{\cos^2(30^\circ) \cdot (m_1 + m_2)^2 + \sin^2(30^\circ) \cdot (m_1 - m_2)^2}$$

$$= \sqrt{(\cos^2(30^\circ) + \sin^2(30^\circ)) \cdot (m_1^2 + m_2^2) + 2 \cdot (\cos^2(30^\circ) - \sin^2(30^\circ)) \cdot m_1 \cdot m_2}$$

$$= \sqrt{m_1^2 + m_2^2 + 2 \cdot \cos(60^\circ) \cdot m_1 \cdot m_2}$$

so we get

Eq. 5e: $$|a_3| = \sqrt{m_1^2 + m_2^2 + m_1 \cdot m_2}$$

And the direction of the acceleration of m_3 is:

$$\tan(\phi) = |a_{3x}| / |a_{3y}| = \frac{\sin(30^\circ) \cdot (m_1 - m_2)}{\cos(30^\circ) \cdot (m_1 + m_2)}$$ so

Eq. 5f: $$\tan(\phi) = \tan(30^\circ) \cdot \frac{(m_1 - m_2)}{(m_1 + m_2)}$$

The general equations for a center of mass are:

Equations 6: general equations for a center of mass $cm = (cm_x, cm_y)$, and the special case of an equilateral triangle

Assumptions 6: same as Assumptions 2 and Assumptions 3

Variables 6:

m_i — masses

x_i and y_i — x and y coordinates

cm_x and cm_y — x and y components of center of mass position

ϕ — the angle from the y-axis of the line from m_3 to the common center of mass

Eqs. 6a: $$cm_x = \sum_{i=1}^{n} m_i \cdot x_i \quad \text{and} \quad cm_y = \sum_{i=1}^{n} m_i \cdot y_i$$

✓ **NOTA MOLTO BENE:** the center of mass of 2 point masses is per force collinear with them, i.e. on the straight line passing through both masses. Note also that if we calculate the center of mass of 2 of 3 masses, then "add" the 3rd mass—*in our special case of an equilateral triangle*—the center of mass of all 3 will be on the straight line passing through both the 3rd mass and the center of mass of the first 2. This means that—in our special case, and also when all 3 masses are collinear (remember L1-L3!), but not in general—the force due to gravity on a 3rd mass is toward *both* the center of mass of the first 2 masses *and* simultaneously toward the center of mass of all 3 masses, neglecting the finite speed of gravity. There very well may be a way to use this result to give a simple yet complete demonstration of "stable equilibrium" (with those "tadpole" and "horseshoe" orbits; see **Figure 5**, **p. 140**), avoiding abstruse or arcane mathematical theory… beautiful though it is. This is worthy of study. This is the kind of insight—here into the dynamics of Trojan points and Trojan bodies—that can come from *freely and critically questioning accepted scientific beliefs*.

The center of mass of all 3 masses is:

Eqs. 6b: $$cm_x = \sin(30°) \cdot \left(\frac{m_1 - m_2}{m_1 + m_2 + m_3} \right) \quad \text{and}$$

$$cm_y = \cos(30°) \cdot \left(\frac{m_3}{m_1 + m_2 + m_3} \right)$$

and just as a check, since we don't really need the value, we will find the angle that the line from m_3 to the common center of mass makes with the y axis to compare it with **Eq. 5f**:

$$\Delta x = cm_x - 0 = \sin(30°)\cdot\left(\frac{m_1 - m_2}{m_1 + m_2 + m_3}\right) \quad \text{and}$$

$$\Delta y = \cos(30°) - cm_y = \cos(30°) - \cos(30°)\cdot\left(\frac{m_3}{m_1 + m_2 + m_3}\right)$$

$$= \cos(30°)\cdot\left(1 - \frac{m_3}{m_1 + m_2 + m_3}\right) = \cos(30°)\cdot\left(\frac{m_1 + m_2 + m_3 - m_3}{m_1 + m_2 + m_3}\right) \quad \text{so}$$

$$\Delta y = \cos(30°)\cdot\left(\frac{m_1 + m_2}{m_1 + m_2 + m_3}\right)$$

$$\tan(\phi) = \frac{\Delta x}{\Delta y} = \left(\frac{\sin(30°)\cdot(m_1 - m_2)}{m_1 + m_2 + m_3}\right) / \left(\frac{\cos(30°)\cdot(m_1 + m_2)}{m_1 + m_2 + m_3}\right)$$

$$= \frac{\sin(30°)\cdot(m_1 - m_2)}{\cos(30°)\cdot(m_1 + m_2)} \quad \text{so}$$

Eq. 6c: $$\tan(\phi) = \tan(30°)\cdot\frac{(m_1 - m_2)}{(m_1 + m_2)}$$

which is the same as **Eq. 5f**. (Yes, we were playing a little fast and loose with the signs of Δx and Δy.)

At least in our special case of an equilateral triangle, this is a reasonably complete demonstration that the gravitational acceleration of each body is in the same direction as both the center of mass of the other 2 and the common center of mass of all 3 (again, assuming infinite speed of gravity).

We would now like to calculate the distance of mass 3 from the common center of mass.

$$r_3 = \sqrt{(x_3 - cm_x)^2 + (y_3 - cm_y)^2} = \sqrt{cm_x^2 + (y_3 - cm_y)^2}$$

$$= \sqrt{\frac{\sin^2(30°)\cdot(m_1 - m_2)^2}{(m_1 + m_2 + m_3)^2} + \frac{\cos^2(30°)\cdot(m_1 + m_2)^2}{(m_1 + m_2 + m_3)^2}}$$

$$= \frac{\sqrt{(\sin^2(30°) + \cos^2(30°))\cdot(m_1^2 + m_2^2) + 2\cdot(\cos^2(30°) - \sin^2(30°))\cdot m_1\cdot m_2}}{m_1 + m_2 + m_3}$$

$$= \frac{\sqrt{m_1^2 + m_2^2 + 2 \cdot \cos(60°) \cdot m_1 m_2}}{m_1 + m_2 + m_3}$$

Eq. 6d: $$r_3 = \frac{\sqrt{m_1^2 + m_2^2 + m_1 m_2}}{m_1 + m_2 + m_3}$$

If we now take the ratio of the acceleration (**Eq. 5e**) to the distance (**Eq. 6d**), we get:

Equations 7: ratio of the absolute value (scalar) of the gravitational acceleration of m_3 toward the common center of mass to the distance of m_3 from the common center of mass

Assumptions 7: same as Assumptions 2 and Assumptions 3, and

3 masses are in an equilateral triangle

Variables 7:

r_i — the *r*adial (linear) distances (between the masses)

m_i — the *m*asses

a_{3i} — *a*ccelerations of mass 3 toward mass *i*

$\dot{\theta}$ — the angular velocity

Eq. 7a: $$\frac{|a_3|}{r_3} = \frac{\sqrt{m_1^2 + m_2^2 + m_1 \cdot m_2}}{\left(\frac{\sqrt{m_1^2 + m_2^2 + m_1 \cdot m_2}}{m_1 + m_2 + m_3} \right)} = m_1 + m_2 + m_3$$

and the symmetry among the 3 masses means that interchanging any 2 still gives us the same ratio:

Eq. 7b: $$\frac{|a_1|}{r_1} = \frac{|a_2|}{r_2} = \frac{|a_3|}{r_3} = m_1 + m_2 + m_3$$

If we combine **Eq. 4c** ($a = v^2 / r = r \cdot \dot{\theta}^2$) and **Eq. 7b** we get:

Eq. 7c: $$\dot{\theta} = \sqrt{m_1 + m_2 + m_3}$$

✓ NOTE: we would really need to deal with the problem of physical units to do all this right.

So we have shown that, as long as we are dealing only with circular orbits around the common center of mass—and by implication zero radial velocities (think polar coordinates with the origin at the common center of mass)—there exists an angular

velocity that will "balance" the gravitationally induced radial accelerations and the rotationally induced centripetal (or "center seeking") radial accelerations of all 3 masses. I.e. we have Lagrange's homographic solution for circular orbits, i.e. with at least "unstable equilibrium" at the Trojan points.

It remains to look at the expansion and contraction of the equilateral triangle.

3.10 Homographic-Equilateral Expansion and Contraction

All that we need for the more general case is to have initial radial velocities be proportional to the radial distances. (Note: they can all be negative and still be proportional to the radial distances.) If the initial radial velocities are proportional to the corresponding initial (radial) distances from the common center of mass/center of rotation of the equilateral triangle (thus the center of revolution of the 3 bodies), and the initial angular velocities are equal, making the initial tangential velocities also proportional to the radial distances, then the changes in the radial distances will be proportional, and the already proportional radial distances will *remain* proportional.

Since the initial radial forces/accelerations are proportional to radial distance, (and since the initial angular accelerations are equal—and non-zero since the radial velocities are non-zero—and thus the tangential accelerations are proportional to the radial distances), then changes in the radial velocities will be proportional to the radial distances, radial distances will remain proportional, and, even with expansion and contraction, the equilateral triangle will homographically remain equilateral. (This really needs calculus to at least feel more rigorous about it, but the arguments would have been considered both simple and cogent enough by Newton and his lesser contemporaries, and even in their style, which is by intention here.)

Thus we have Lagrange's full homographic solution, i.e. that there exist initial conditions which will allow homographic expansion and contraction of the equilateral triangle formed by the 3 point masses as it revolves around their common center of mass (barycenter). We perhaps even one up him by considering the case of the angular velocity being 0 and the radial velocities all being greater than their respective escape velocities, so the equilateral triangle just expands to infinity without rotating.

3.11 Trojan Points?!

Thus we have demonstrated the existence of Trojan points using only algebra, trigonometry and Newton's law of gravity. Or have we?! Actually we have demonstrated a purely theoretical result, that 3 gravitational bodies (arbitrary "point" masses) in an equilateral triangle are "stable" in this configuration even when the triangle is rotating and expanding and contracting, *if* they are not "perturbed" by other "forces". We have not yet shown that they will be in a "stable equilibrium" in the usual sense, i.e. that they will tend to "return" to—or "stay close" to—the equilateral configuration if they are "perturbed" away from it, or if they were not initially in this configuration but "close" to it. Actually, "return" is not strictly correct since the asteroid or other body will in general tend to *orbit* the Trojan point, *not* return to it, which means that a good bit of the time it may seem to be "unstably" moving away from the Trojan point.)

My original intention was to provide such a simple demonstration of the existence of "unperturbed" Trojan points, Lagrange's L4 and L5, that even Newton's lesser contemporaries would be ashamed not to have "discovered" it. This I feel I have accomplished. But I also held onto the hope that I would be able to provide a similarly simple demonstration of the "simple stability" of L4 and L5, but I have so far failed to do so.

So this is where Lagrange's perturbation theory comes in: so far, only it shows with reasonable mathematical surety that Trojan points have the ability to trap and keep bodies with small enough Trojan-point-relative velocities in "close" orbits; i.e. their relative velocities must be below a "stability escape velocity", much the same as with rockets and satellites. We need to remember, however, that Lagrange did not predict the shapes of the tadpole(s) and horseshoe orbits that we now know exist. He had nothing like the computing power of a turn of the millennium laptop with MathCAD 2000 (yes, I have been slogging away at this as one of *many* projects for a *long* time) which I used to generate **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, let alone access to a large mainframe/supercomputer that would ordinarily be used to plot the tadpoles and their conjoining horseshoe. All he could comfortably predict was "simple stability", which he obviously knew meant that L4 and L5 could be "centers" of orbiting "Trojan Planets".

To be fair we will note that one could do a simplified version of perturbation theory, by *guessing* (we don't know ahead of time e.g. what the size/shape/etc of a region of stability around the Trojan point might be, so *guessing*) at certain particular small perturbations to be applied to the 3 bodies as they approximately orbited their barycenter, and calculating what then happens to all 3 bodies. I.e. we could try small perturbations of one or more of the bodies in various directions with various velocities away from "its Trojan point" (it is essential to remember that *each* of the bodies is in either L4 or L5 with respect to the other 2 bodies, not just the "tiny guy"), and see if it will keep wandering away, or circle back, or whatever. This is actually what perturbation theory does, but employing much higher caliber—and much more arcane *and* computationally intensive—mathematical guns. But, of course today we know that this could take quite a bit of computation, e.g. if we happened to set up a "horseshoe perturbation" as our initial guess.

In fact, this is more or less what I did, but with the "brute force" of a computer, the results of which can be seen in **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, **Figure 6a: Are Stable Trojan Star Systems Possible?! Part a, p. 142**, and **Figure 6b: Are Stable Trojan Star Systems Possible?! Part b, p. 144**. The 2 large masses were placed along a horizontal mid-line around the middle of the contour plots. Then the third was placed at each point in the contour plot (except the mid-line, where the calculations would divide by 0). This, of course, is a lot more than just "a few small perturbations". Then, using the falling rate difference, the tendency for all 3 masses to form a *more* equilateral triangle was computed for each point, and color contour plotted. The thing left out was any *initial movement* of any of the 3 masses; i.e. this is really a static, 2-dimensional result, not the full dynamic result that it should be. That would need 3 dimensions for the changing positions of the 3 bodies, and *at least* 3 more "degrees of freedom" for the perturbed velocity of *at least* the "infinitesimal" body. But, the computations for the static case, since they are based on the falling rate differences of the 3 masses, are *wonderfully* easier than trying to use Lagrange's perturbation theory or partial differential/difference equations to calculate more or less the same thing!

3.12 Stability?!

So, we still have the question of orbital stability in relation to Trojan asteroids. As **Figure 3: The 3 Masses and 2 of Their Centers of Mass, p. 138**, shows, the alignment of the top mass (m_3) with the center of mass of all 3 (the barycenter) and the center of mass of the other 2 masses (in that order) holds for *all 3 masses*. These alignments are essential to maintaining the static equilateral triangle sans

perturbation(s). The gravitational forces that they experience in the static triangle (not necessarily equilateral or isosceles) are simply combined with the "centrifugal type forces" that arise from their movements. But what can we say about perturbations and how they affect our equilateral triangle as it rotates, perhaps expanding and contracting?

- We can try to use our knowledge that when the lighter and heavier bodies are in an isosceles triangle with the Earth, with the lighter one *leading* (near L4 with respect to the other 2), the lighter will tend to fall/accelerate faster toward the Earth when the angle with Earth as the vertex is less than 60°. (See **Figure 2: The Difference in Falling Rates as a Function of Angular Separation, p. 137**.) In that case the radius of its approximate orbit around the barycenter would relatively decrease, putting the lighter one into a relatively faster orbit and making it speed up relative to *its* L5 body, widening the angle back toward 60°. If the angle is greater than 60°, then the following, heavier body will tend to fall faster toward Earth, making itself speed up to close the angular separation back toward 60°. So far, so good.

  (We are here ignoring that when all 3 bodies are co-orbiting, this "falling faster or slower" must be understood differently. This needs more mathematics to do right.)

  But we also have to consider what would happen if the heavier body were leading (in L4). In that case, the heavier one will tend to fall faster if the angular separation is greater than 60°, making it relatively speed up, *increasing* the angular separation. If the angular separation is less than 60°, the following, lighter body will fall faster and into a faster orbit, decreasing the angular separation yet further. This seems to be an "oops!", but is it?!

  This definitely does not suggest the stability we are looking for, that we know is there both from Lagrange's theory and from astronomical observation. But, we have not taken all the forces and movements into account, in particular that the orbits that can be faster or slower are really approximately around the barycenter and not around the center of mass of the most massive body (ala Kepler), and if this is a smaller effect than the stability induced by whatever actually does provide the empirically known stability, this effect just described *does* help explain the observed asymmetry in the relative numbers of Trojans in the leading and following tadpoles (more on which later); noticeably more have been observed in the leading L4 tadpole than in the following L5 tadpole. As far as I am aware, Lagrange did not predict that the leading L4 Trojan asteroids would outnumber the following L5 asteroids, which is what has so far been found by telescopic observation. Neither do I know of anyone else who has advanced an explanation for the relative numbers, so this is a first. (See **Section 5.2, Trojan Point Astronomy in the 20th and Early 21st Century, p. 71**.)

This analysis, albeit on the simplistic side, does not indicate why Lagrange's perturbation theoretical analysis of Trojan points found the requirement for "stability" with "perturbations" that 1 of the 2 non-infinitesimal bodies be of an "intermediate infinitesimality"— $m_2 < \sim 0.04\ m_1$ (where, for Lagrange, m_1 was the Sun and m_2 was Jupiter). It would require a bit more careful analysis to satisfyingly resolve whether Lagrange's requirement was merely an artifact of his method and not a fact of nature. This kind of thing (artifacts e.g. of method/methodology) does happen in science, and is in general a neglected study, given its fundamental importance.

Too, there is the question raised by **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, concerning whether L3 is a "stable" point like L4 and L5, or whether it is "unstable" like L1 and L2. Lagrange held, as astronomers still hold, that L3 is "unstable", so that it could not hold that mythical, mirror-image Earth, invisible to us on the opposite side of the Sun. But the green color coding of this region of the contour-plot, just as the green regions around L4 and L5, indicates that it is a region of relative "stability" in a larger sense. A (static) point mass in that region would tend to be attracted back, or spit back, into an equilateral triangle with either L4 or L5, whichever was the closer. This makes sense given that L3 is part of the larger horseshoe "orbital well", the entirety of which must be considered part of the "stable" Trojan "orbital well". (I am here coining the term "orbital well" since it's a gravitational potential energy well, but not truly a standard "gravity well", the common—but less general—term for the gravitational potential energy "well" surrounding a—usually large—mass.) The situation must be studied from the standpoint of the entire horseshoe being a "shape of stability", "transcending", above and beyond ("hail the goer"), the concept of "point of stability". All this needs careful study, as further evidenced by the fact that, as of November 2011 (I just found out!), astronomers are still not aware that the horseshoe orbits are as much a Trojan phenomenon as are the tadpole orbits. (More on this in **CHAPTER 5, TROJAN ASTRONOMY IN THE 20TH AND EARLY 21ST CENTURIES**, **p. 73.** Also, when this e-book gets a second edition, I will try to present satisfying resolutions to some of these questions, and provide the *simple*—in terms of Newton's time—arguments for "stability" that I hope-feel are lurking nearby.)

- **L3 Stability (1):** Here is chance for you to check whether you were really paying attention when you read the previous paragraph. The green color coding means that in, this *static* case, the mass indicated in **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, when placed at that point in that region has, *not* a higher probability of moving back toward the center of that particular green region, but *instead* a higher probability of moving back toward an equilateral triangle with the other 2 masses! I.e. it has a higher probability of moving back toward L4 or L5! The green region of L3 seems *un*stable as regards itself because it tends to push the mass there *away* from L3 and *back toward* the stability of L4 or L5. This helps us understand even better how the dynamics of the tadpoles merges into the dynamics of the horseshoe.

  This green region L3-L4-L5 horseshoe stability issue actually starts/helps to raise the *much* larger question: "just what *is* stability?!" (See **L3 Stability (2)** on **p. 130.**)

- This might seem a bit digressional, but is very important to the simple Trojan physics-mathematics presented in this e-book. There is an unusual group of ~ 700 known asteroids called the "Hildas", lying even more strictly within the orbit of Jupiter than do the Trojans. Also interestingly, they all remain much closer to the plane of the ecliptic than do the Trojans, with an estimated half the average inclination (and I personally think more like a third; see http://en.wikipedia.org/wiki/Hilda_family.) They are in a 3:2 resonance with Jupiter (meaning they revolve around the Sun an average of 3 times for every 2 times Jupiter does, unlike the *average* 1:1 resonance Trojans). The interesting part for us here is that, unlike the Trojans with the Sun and Jupiter, the 3 main Hilda subgroups (3 "bunches") form an *almost*

precisely equilateral triangle formation, with the 3 vertices (the 3 "bunches") having much greater numbers of asteroids than the 3 sides (*roughly* linear), and therefore much greater average mass than the asteroids on the sides. I.e. they form an unusual yet distinctly Trojan group with *roughly equal* masses at the vertices of their equilateral triangle (counting the *average* of all the asteroids in the vertex clumps at any one time. In addition, the asteroids apparently migrate around the triangle quite a bit)!

This has an important meaning for us here since Lagrange's Trojan work using Perturbation Theory predicts the Trojans and allows us to study various aspects of their dynamics, but it does not predict the Hildas, nor does it directly facilitate their study, since Lagrange's Trojan results had the restriction that the second largest mass must be a small fraction of the largest mass; i.e. $m_2 < \sim 0.04\ m_1$. Clearly the Hildas do not meet this requirement, at least not on average. That means that the simple approach to Trojan dynamics we have been studying here may apply quite importantly to the Hildas in a way that Lagrange's work can not, e.g. concerning the maintenance of their equilateral triangle despite roughly equal average masses at the vertices of the equilateral triangle! And, despite the 3:2 resonance with Jupiter, the Hildas seem to have essentially intimate interactions with Jupiter's L3! as well as Jupiter's L4 and L5, the 3 most important points/sections of Jupiter's Trojan horseshoe orbit. I have not yet found estimates of the Hilda's average masses at the vertices and along the sides, so if any of you know, let me know. Also, the asteroids migrate from the vertices, along the sides, to the other vertices, but again, I do not yet have much detailed info on how that works. For example, does each Hilda keep making "grand tours" of all 3 vertices? Do they *all* go in one direction? Clockwise or counterclockwise? If they go in both directions, does each always go in the same direction, or do they vacillate? The details must be fascinating!

The host of quote-marks used just above (before the extended remark) are to indicate that our whole concept of such "stability"—in theoretical fact needing to be acknowledged as a "fuzzy" concept—has started to change drastically since Lagrange. He predicted that the Trojan asteroids would "stay close" to the Trojan points, but he did not mention that they could be "relatively stable" in the exaggerated tadpole orbits and even more exaggerated horseshoe orbits that are now ever more frequently being found and studied (even though, as of November 2011, astronomers still have not appreciated that horseshoe orbits are a Trojan phenomenon along with tadpole orbits). That seems to require significant computer analysis of a careful kind. Astronomers now consider asteroids to be orbitally stable even when they are in such exaggerated tadpole orbits, but they must now be considered orbitally stable even when they are in the even more exaggerated horseshoe orbits.

And astronomers are also studying asteroids that *somehow* enter such an orbit, bounce around in e.g. the tadpole for a few thousand years, then *somehow* exit. (There is still magic in that entry-exit processes that has not been understood satisfactorily.) They, too, are now considered to be orbitally "stable", at least when they are "*temporarily in*" such an orbit.

- We seem to be slowly evolving (back) toward a concept of "stability" as being a function of the body in question not having "enough escape velocity" (*in the right direction at the right time*; see next comment). But hopefully,

when we continue to arrive at the place from where we started, we will know the place somewhat better each first time we arrive there.

- ➢ Also, our concept of "stability" is starting to relate ever more to the dynamics of (e.g. the time varying shape of) the "entrapping" (gravitational potential energy) orbital wells, since such orbital wells may well have something equivalent to "(mountain) passes" or "doors", almost certainly *dynamic* "doors" that may be either more "open" or more "closed" at certain times and "places" (varying with the varying distribution of gravitational potentials from other masses) so that only under very special and as yet poorly understood conditions can asteroids enter and become "trapped" ("stable"), and then thousands of years later "escape" (uhh… become "unstable"), when perhaps another such "door" again "opens up *enough*" at just the right time and place (etc). Of course, as they "enter" and "bounce around" inside the orbital wells, the asteroids can easily be gaining or losing velocities (possibly escape velocities) due to "slingshot" effects (and other "perturbations", from either inside or outside the orbital wells) such as those we used to speed our Voyager 1 and 2 spacecraft through and out of our solar system. So the question of "stability" as determined by entrapment and/or escape will be largely determined by an as yet undetermined combination of "doors" (not yet actually known to exist) opening and closing, and gains and losses of energy due to "perturbations" (known to exist) either within or without the tadpole and horseshoe orbital wells.
- ✓ If you were on your toes you probably thought of quantum tunneling and how it relates to our concept of stability.

Our whole concept of orbital stability is—or should be—currently changing-evolving rapidly. This may open the door to a renewed study of rejected possibilities, even of the previously "scientifically" ridiculed as impossible because "unstable" "alternate/mirror image Earth theory" (the "alternate Earth" that remains permanently invisible—from the Earth—since it's opposite the Earth on the other side of the Sun at L3; see **Figure 4: The Lagrangian Points L1-L5, p. 139**, **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, and the **APPENDIX, p. 129**). This possibility is of course more mythically-humorous than astronomically likely since for one thing the gravitational effects would almost certainly have been seen already in the orbits of Venus and Mercury (that's how Neptune was discovered, by its effect on the orbit of Uranus), but studying it with the new eyes of a newer "fuzzier" concept of "stability" just might yield new scientific insights! (Actually, there is another more intriguingly detailed reason why that particular "alternate Earth" would not be "stable" as we currently conceive "stability". See **L3 Stability (1)** on **p. 64** and **L3 Stability (2)** on **p. 130**.) After all, the same people who have ridiculed this idea have also "… oversighted" the falling rate difference of lighter and heavier bodies for over 300 years.

A possibility that is more likely, of course, is that e.g. "relatively stable" Trojan-ternary star systems with approximately equal masses (i.e. not "hierarchical" in the sense of Gérard Henri de Vaucouleurs (1918-1995), the French astronomer; see http://adsabs.harvard.edu/full/1972IAUS...44..353D) will be accepted theoretically, and eventually discovered (i.e. "Trojan", but not strictly "Lagrangian"; i.e. even ones not within the limits/restrictions calculated by Lagrange). (See **Figure6a**, **p. 142**, and **Figure 6b**, **p. 144**, and **Section 5.3, The Future of Trojan Point Astronomy in the 21st Century, p. 73**.)

4 TROJAN POINTS AND THEIR TADPOLE AND HORSESHOE ORBITS

4.1 Trojan Asteroids in Tadpole Orbits

(See also **APPENDIX, p. 129**.) No asteroids are known to be equilibrated precisely at either Trojan point—L4 or L5—proper. Think of viewing the arrangement of masses from "above", rotating our view of the movements by keeping the 2 larger masses—here the Sun and Jupiter—on either a baseline at the bottom of our view (of the leading Trojan point L4) or a "topline" at the top of our view (of the following L5). Viewing them all in this way, the asteroids can be seen to tend to orbit the Trojan point in an orbit that is elongated, non-elliptical, and not even symmetrical.

Each tadpole's "orbital well" (see **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**) bends so that its center of curvature is roughly the center of mass of the whole ensemble (near the largest mass, e.g. the Sun), its "rounded head" curves so that it points toward the 2nd largest mass (e.g. Jupiter, which is roughly 1/1000 the mass of the Sun), and its "pointy tail" curves around toward L3. At some point, someone decided it was shaped like a "tadpole", and that's what these "orbital wells" are now called. **Figure 5**, **p. 140**, a contour plot derived from a simple function of the falling rate difference (for the static case), gives a rough idea of the shape. It can take hundreds of years for one of Jupiter's Trojan asteroids to complete a tadpole orbit of its Trojan point.

- It is important to note that, although they probably do (except for local gravitational interactions), the paths of Trojan asteroids may not actually follow the outlines of the tadpoles very closely. Those shapes mark out a potential energy level "topo map" of the orbital well in which the Trojans move. What is generally known of orbital physics says that if the asteroid is closer to the Sun than is (e.g.) Jupiter, then it will be advancing in its orbit (going counter-/anti-clockwise with respect to Jupiter, i.e. revolving faster than Jupiter around the Sun), and if it is farther away from the Sun, then it will be retarding in its orbit. This strongly suggests a general "clockwise" circulation pattern of the asteroids within their tadpoles, looping around L4 or L5, respectively (or around both in the horseshoe). I know of no study, especially any complete and detailed study, of how they actually "circulate", i.e. "bounce around" inside the orbital well. For example, I have found no mention of Trojans that have a "retrograde"—counter-/anti-clockwise, viewed from above—orbit in their orbital wells. Nor have I found any indications concerning whether *any* are known that, as opposed to merely "circulate", "bounce around" in some interesting fashion inside their orbital wells, perhaps partly because of local gravitational interactions.

4.2 Trojan Asteroids in Horseshoe Orbits

(NOTE: as of November 2011, I just found out, astronomers are still not aware that horseshoe orbits are a Trojan phenomenon along with tadpole orbits.) **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**, also shows how the tadpole orbits for L4 and L5, which are mirror symmetrical, start to "grow" or extend around to the other side of the more massive body until they blend into a "horseshoe" shaped orbit. Asteroids are actually known to move in these horseshoe shaped orbital wells so that they orbit *BOTH* Trojan points! They have enough kinetic energy in the proper direction to escape the tadpole orbital well, but not enough to escape the larger

horseshoe orbital well. Although it is possible that we might someday find one in a horseshoe orbit, all the known Trojan asteroids of Jupiter seem to be restricted to their respective tadpoles, but with more found in the leading tadpole around L4. This is probably a clue to something important, but what?! (See comment on **p. 63** for a possibility.) **Figure 5**, **p. 140**, suggests that there is a lot more to learn about Trojan points and their tadpole and horseshoe orbits.

The Earth "companion" asteroid, 3753 Cruithne (as of November 2011 still not recognized by astronomers as a proper Earth Trojan), is in a very unusual, Earth relative horseshoe orbit, one that is inclined at an extreme angle to the orbit of Earth around the Sun. It takes about a year for Cruithne to complete its circuit of a bloated tadpole (roughly kidney bean shaped) sub-orbit, but the bloated tadpole orbit moves slowly from one end of the horseshoe, near Earth, "around Cape Horn" (in irrelevantly nautical imagery) to the other end of the horseshoe, again near Earth. Cruithne's full horseshoe orbit takes about 770 years to complete and occasionally takes it almost directly *above* Earth (over the ecliptic). This indicates that the 2-dimensional study seen in **Figure 5**, **p. 140**, does not give anything like the whole picture, by any means. In fact, this orbit seems to be a combination of a modified horseshoe orbit with "retrograde satellite motion"—one of several new classes of co-orbital motion found recently (1999) by Fathi Namouni, Apostolos Christou, and Carl Murray of Queen Mary and Westfield College in London (and probably other colleagues)—in which an asteroid slowly orbits a planet at a great distance, perhaps even half the distance between the planet and the Sun.

(For more on 3753 Cruithne, see http://focus.aps.org/story/v4/st16 and http://prola.aps.org/abstract/PRL/v83/i13/p2506_1 Fascinating!…)

- ✓ For those who are concerned with the very important concept of orbital resonance, we can note that Trojan bodies are considered to have an orbit with 1:1 resonance with the body whose L4 and/or L5 they are orbiting (e.g. Jupiter or Earth), meaning here that they make the same number of orbits in the same amount of time… approximately, on average. Any body in a tadpole or horseshoe orbit will go faster than 1:1 for a while, then it will reverse and go slower than 1:1 for a while. The long term *average* will of course asymptotically approach 1:1, as long as the body *stays* in the tadpole or horseshoe orbit. That orbital resonances can be important is shown in the examples that unstable resonances with Jupiter have led to the Kirkwood gaps in the asteroid belt between the orbit of Jupiter and Mars, and also unstable resonances with its moons have led to breaks in the rings of Saturn.
- ➢ We have a further question related to the homographic property that Lagrange showed for the "geometrically pure" equilateral triangle arrangement. Can the theoretical homographic property of the pure equilateral triangle points, L4 and L5, be found in actual Trojan asteroids in their tadpole and horseshoe orbits? Do they/their orbits have a way of expanding and contracting that we could call "strictly homographic"? Or do their distances from e.g. Jupiter and the Sun affect the "geometric purity" of the homographic expansion and contraction that must happen to *some* extent as e.g. Jupiter and the Sun orbitally move between their perihelion and aphelion, which differ by about 75 million kilometers, every 11.86 years?!

4.3 Trojan Space Debris?! Trojan Atmospheres?!

Even though we are anticipating somewhat the discussion in **Chapter 5, Trojan Astronomy in the 20th and Early 21st Centuries, p. 71**, it makes sense here to look

at some of the further possibilities for what might happen in Trojan tadpole and horseshoe orbits since these relate to the dynamics under discussion.

We know that there are hundreds—and probably thousands—of Trojan asteroids large enough to see with good amateur telescopes associated with Jupiter's L4 and L5. It should be obvious that low relative-velocity bodies of much smaller size will also tend to become trapped in the "concentric" tadpole orbits, which form a valley surrounding a Trojan point, so that they orbit the Trojan point. Viscosity or drag from the tenuous Solar System atmosphere and from other "space debris" will cause these orbits to decay extremely slowly, but fairly surely.

Perhaps this needs further explanation. Despite the fact that we think in terms of "empty space, there is actually an atmosphere (non-zero partial pressure, though possibly so near zero as to be "negligible" in the short term) and even much "space debris" in our Solar System. Matter comes from the solar wind (that flows more-or-less radially outward from the Sun, then whatever didn't have escape velocity and hasn't been captured by some planet, asteroid or whatever, including possibly a Trojan "orbital well", falls more-or-less radially back toward the Sun), and there is a whole range of plasmic, atomic, molecular, and particulate matter ranging up to official asteroid and even planet size, that yields a small but effectively non-zero viscosity.

This viscosity-friction could act over extremely long periods of time to slow even larger asteroids, so their orbits around a Trojan point inevitably decay. This atmosphere and this space debris is also in some orbit or other, so it may in fact be traveling with a Trojan body and not affecting it viscously, but there will tend to be much more in some other orbit that does. The collisions will by and large not be perfectly elastic, far from it, and even if they are, the orbits of larger objects will tend to eventually decay entropically. However, bodies will from time to time be ejected when given enough energy by slingshotting off the other bodies there to attain escape velocity, leaving the bodies with which it interacted with lower energies *and* further decaying orbits. And perturbations from planets, too, could cause bodies to escape the tadpoles or horseshoe.

The tadpole orbits around Trojan points will not only tend to trap low energy space debris, they are *perhaps* even capable of maintaining a non-negligible atmosphere. A low temperature, low pressure atmosphere will tend to collect and be held there, even without the actual presence of a larger gravitational body at the Trojan point proper, or orbiting in the tadpole. So the study of Trojan bodies overall is far from simple, especially when we take 3 spatial dimensions into account as we must with the asteroid 3753 Cruithne (see **Section 4.2, Trojan Asteroids in Horseshoe Orbits, p. 67**).

It is almost certainly worth sending space probes to the Trojan tadpoles of the Earth-Moon system (and the Sun-Earth tadpoles, as well). They are (more or less) the same distance away as the moon, extremely close by space exploration standards, much closer than the asteroid EROS to which we sent the NEAR space probe at a cost of $224 million. (The NEAR spacecraft touched down on asteroid Eros on Monday, February 12, 2001. The Near Earth Asteroid Rendevous mission was renamed NEAR-Shoemaker in honor of the late astronomer, Gene Shoemaker. For more, see http://near.jhuapl.edu/) Trojan point studies could probably be done with noticeably less expense. The atmosphere could be studied, and debris would be concentrated and much easier and cheaper to find there and bring back to Earth than in/from space in general, even than from the Moon.

UPDATE: it turns out that quite some time ago someone independently suggested much this same idea. In 1951, Professor J. Witkowski predicted the existence of a

photometrically confirmable concentration of dust at Earth's L4 and L5 Trojan/-libration points. Although the existence of these clouds is still disputed, such clouds were ostensibly first seen by Kazimierz Kordylewski in 1956. Between March 6 and April 6 of 1961 he succeeded in photographing two bright patches near Earth's L5.

In 1967, J. Wesley Simpson made observations of the clouds using the Kuiper Airborne Observatory. The Japanese Hiten space probe (1990), which passed through the libration points to detect trapped dust particles, did not find an obvious increase in the density of dust levels above that in surrounding space. This is consistent with the observed low densities of particles in some of Saturn's visible rings. Further such studies are suggested far all Sun-planet systems and their planet-moon systems.

The reader is referred to http://en.wikipedia.org/wiki/Kordylewski_cloud and to http://www.rfreitas.com/Astro/SearchIcarus1983.htm for all this and more.

5 TROJAN ASTRONOMY IN THE 20TH AND EARLY 21ST CENTURIES

5.1 Historical Digression—Why "Trojan" Points?

Lagrange apparently called the astronomical bodies he predicted "Trojan Planets" because there was a hidden quality to them, reminiscent of the (Greeks hidden in the) Trojan horse of Homer's *Iliad*. This hollow wooden horse was constructed at the suggestion of the crafty Odysseus to seem to be a sacrifice to Athena. After it was constructed outside the walls of the city of Troy, the Greeks appeared to leave in their ships, "defeated". The horse actually hid the Greek warriors inside from the Trojans who took the horse into their city to crown their victory over the Greeks, even though the cursed prophetess Cassandra warned them not to. We can note with further irony that the curse on Cassandra caused people not to believe her. The Greeks came out of the horse at night when the people of Troy were asleep and unlocked the city gates to let in the rest of the Greek warriors who had returned under cover of night. Together they all sacked and slew the city and, "among other things", gave rise to "the sequel", Virgil's tale of the Æneid and the founding of Rome, and a lot of "action figures".

The "hidden" aspect of the Trojan points has to do with a characteristic of Lagrange's conception of the astronomical problem. When Lagrange developed perturbation theory and found the (theoretical) Trojan points, he was mathematically looking at the Sun, at Jupiter which was very much smaller (approximately 0.1% of the Sun's mass, "infinitesimal" relative to the Sun), and at what would happen if a body infinitesimal with respect to Jupiter were perturbed from its position in a Trojan point leading or following Jupiter. Since any such bodies would be small, they would be invisible by the astronomical standards of his day, "hiding" where they were not expected, so he called them "Trojan points" after the Trojan horse (hiding the Greeks). Or so I vaguely remember reading a long time ago.

This is more than historically interesting since it brings up an important difference between the approach explored here and Lagrange's perturbation theory (based on partial differential equations and his calculus of variations). Lagrange was not thinking in terms of arbitrary non-infinitesimal and even potentially equal masses at each vertex of the equilateral triangle. And astronomers since have not, either. Astronomy is extremely subject to the psychological reality that if you don't know where to look, or don't know what to look for or how to look, or especially even *to* look, then it is very easy to remain blind to even the most obvious events.

5.2 Trojan Point Astronomy in the 20th and Early 21st Century

It was February 22, 1906, more than a century after Lagrange, that the astronomer Max Wolf—also credited as the first astronomer to use photography to do astronomy—finally proved that Lagrange was correct 134 years earlier by finding the asteroid 588 Achilles (perhaps the most famous Greek warrior-hero in the Trojan war) near the leading Lagrangian point of Jupiter, L4 (soon after designated as the "Greek camp/node"). (More recently, unofficial credit has been assigned to the American astronomer Edward Emerson Barnard for his 1904 sighting of a Trojan, only re-discovered in 1999, but unfortunately he failed to realize what he had discovered, so the official credit stays with Max Wolf.) Within a year August Kopff had found 617 Patroclus (a Greek) near the trailing point, L5 (the "Trojan camp/node"), and 624 Hektor (a Trojan), again near the leading point (the "Greek camp/node"). Interestingly, Patroclus has a companion, Menoetius. They are a binary (pair), revolving around

each other, once considered rare for Trojans (see end of this section), and both are thought to be ex-comets. Astronomers currently (2008) think that most of the Trojans are comets that originated in the Kuiper belt and were captured by Jupiter in the early stages of solar system evolution.

After the first few were discovered it was decided to name all Trojan asteroids (which Lagrange had called "Trojan planets") after the heroes of the Trojan war, with the leading point bodies named after Greek heroes and the following point bodies named after Trojan heroes. All, that is, except the Greek Patroclus in the trailing Trojan group and the Trojan Hector (sometimes written Hektor) in the leading Greek group. They are today considered "spies" in the others' camps.

- If you are trying to remember, it was the Trojan Hector (ancient Greek: **Ἕκτωρ**, Hektōr, "holding fast"), son of King Priam (ancient Greek: **Πρίαμος**, Priamos), who killed the Greek Patroclus (ancient Greek: **Πάτροκλος**, Patroklos), an archetypically beloved friend (and cousin by some accounts, depending on the mother accounted) of Achilles (ancient Greek: **Ἀχιλλεύς**, Achilleus), after which Achilles started fighting again and, as the war came to a close in its tenth year, finally killed Hektor. (And don't forget Helen—ancient Greek: **Ἑλένη**, Helénē—and… "We'll always have Paris—ancient Greek: Πάρις!")

These asteroids all slowly orbit their respective Trojan points in relatively stable non-elliptical tadpole orbits. Jupiter's Trojan asteroids can take hundreds of years to complete such an orbit. (See **APPENDIX, p. 129**, and **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**.)

When the basic thesis of "Newton's Great… Oversights" was first written in 1995, and published on the Internet, there had only been minor interest in Trojan bodies, and less for non-Jupiter Trojans. Although theoretically Trojan-style planets, moons or asteroids could exist in the orbit of any planet or moon, no exhaustive search for them had been performed, and this despite the fact that searching Trojan points for asteroids is an obviously easier study than searching for asteroids in general (much smaller volumes of space to examine).

In 1990 Dr. Hannes Alfven (1908-1995), Nobel Laureate in Physics (for "contributions and fundamental discoveries in magnetohydrodynamics"), was still suggesting that such asteroids do exist in the orbit of Earth, both leading and following Earth, to encourage such searches. And in that same year the American astronomers David H. Levy and Henry E. Holt reported finding the first Trojan asteroids in the orbit of Mars. In 1999, the Earth companion 3753 Cruithne with its very strange but still recognizably horseshoe-type Trojan orbit was discovered (see **Section 4.2, Trojan Asteroids in Horseshoe Orbits, p. 67**). In fact, the discovery of Cruithne may have been the turning point in public interest in Trojan bodies with their tadpole and horseshoe orbits.

By 2005, the dearth of interest in searching for Trojan bodies had decidedly waned. But now Trojans are again being enthusiastically searched for and rapidly discovered. Although as of 1999 fewer than 200 had been numbered, as of July 2004, ~ 800 Jupiter Trojan asteroids had been named or numbered, and somewhat studied, and as of August 2007, 1179 (640 named and 539 numbered) have become official near L4 and 1045 (536 named and 509 numbered) near L5. But there are estimates of thousands of Jupiter Trojans with diameters greater than 15 kilometers, always more in the leading group than in the trailing group. (For more, see http://en.wikipedia.org/wiki/Trojan_asteroid ; Wikipedia generally offers lots of good

info; it may be less accurate than Britannica—to which it is often compared—in some ways, but new material can be added *very* quickly.)

Currently, Trojan asteroids are known to exist not only in the orbit of Jupiter, but in the orbits of some of Jupiter's moons, Saturn's moons, and even in the orbits of Mars, where Eureka was discovered in 1990 near L5 (and several others since), and of Neptune. Saturn's moon Dione is accompanied by Trojan sibling moons: Helene at its L4 point and (tiny) Polydeuces at L5. Saturn's moon Tethys is also accompanied by Trojan sibling moons: Telesto at L4 and Calypso at L5. Saturn's Epimetheus and Janus, of roughly equal diameter and mass, are "co-orbitals", doing a Trojan horseshoe orbit dance with each other. As of 2008, however, the Minor Planet Center which operates at the Smithsonian Astrophysical Observatory has not recognized any asteroid as being an official Trojan of Mars.

It turns out that "many" Trojan asteroids are binary, i.e. they come in pairs that orbit each other. Some are even "contact binaries" which orbit each other so closely and slowly that they remain in physical contact. (See Mann, Jewitt, and Lacerda, "Fraction of Contact Binary Trojan Asteroids", The Astronomical Journal, 134:1133–1144, 2007 September, the abstract of which can be found at http://www.iop.org/EJ/abstract/-search=56910649.1/1538-3881/134/3/1133)

Here I apologize to you, Gentle Reader, because I stopped trying to keep the above (strongly) up to date in roughly 2005. Interesting discoveries have been made since then, e.g. at least several Sun-Neptune Trojans, for which I direct you to Wikipedia. (See http://en.wikipedia.org/wiki/List_of_objects_at_Lagrangian_points.)

UPDATE: I *finally* became aware, in November of 2011, that a new Earth Trojan had been discovered, the first to be found in a tadpole orbit. It is still only known as "**2010 TK7**". Please see:

http://www.nature.com/nature/journal/v475/n7357/full/nature10233.html

Simultaneously I was made aware that astronomers have heretofore not noticed that horseshoe orbits are really just a natural extension of tadpole orbits.

5.3 The Future of Trojan Point Astronomy in the 21st Century

The physics of Trojan points gives us the dynamics of those fascinating "horseshoe" and "tadpole" shaped orbits of asteroid size bodies relative to Jupiter, the Earth and other planets, and of Jupiter's moons. (See the *Encyclopedia of the Solar System*, Weissman, McFadden, Johnson, Eds.; Academic Press, 1999, pp. 815-7.) The new and very simple approach to Trojan point physics and dynamics given here—even if it is only 2-dimensional—could conceivably yield new insights, but at the very least it can help stimulate popular interest in the physics of these fascinating astronomical phenomena. It is hoped that this can contribute to advancing our knowledge of Trojan bodies and their place in the formation of the Solar System, especially since the mathematics presented in this book is so simple that it is amenable even to high school students with modest home computers.

Computers open up new worlds of possibilities, even to an important extent to amateur astronomers and physicists. There are 2 broad and overlapping classes of problems that computers can help even amateur physicists and astronomers with:

- **Study of existing Trojan bodies:**
  We can hope that simple but effective software and the appropriate astronomical data can eventually be made generally available to those amateurs and amateur-professionals ("moonlighting" in their "free time") who would like to study existing Trojan bodies and their dynamics. This could greatly help in the search for and determination of new Trojans.

- **Study of generic Trojan systems and their dynamics, make that generic astronomical systems and their dynamics:**
  We need to study how Trojan (and Hilda) systems work—and don't work—in general. Computer simulations and other computer-aided studies can help give us more of a "gut feel" for how they actually work. Generic studies could help us understand how generic Trojan systems develop and interact over time with their non-Trojan "neighbors" (including planets and wide ranging orbit crossing asteroids and comets). This could help form a basis for "what-if" studies of actual or imagined Trojan systems, e.g. "what-if the masses don't adhere to Lagrange's requirements?"

  We need to reiterate here that we still tend to assume that Lagrange's work on the dynamics of his own Lagrangian points is basically unflawed, and this it may be. But, since Lagrange, like Newton, overlooked that lighter and heavier bodies fall at different rates, we must ask and investigate: what else may Lagrange have overlooked?! This should not be a dismaying question. In fact, it should inspire many to renewed research into Lagrange's work and possible extensions in addition to possible "… oversights".

Computers can be better utilized to carefully study analytically intractable problems that have no closed form solutions, the deceptively "simple" 3-body gravitational problem being a classic example. Lagrange made computationally convenient but over-simplified assumptions that are not physically realistic, such as the "infinitesimal" 3rd body. The general solution to even the limited 3-body problem would have been computationally—besides analytically—intractable for him, *in his day*, well before computers, if he had *not* done so. Over time, assumptions such as Lagrange made and the results or predictions extrapolated from them have a way of diverging from reality, especially since our observational abilities tend to keep improving, if fitfully. Computers can be used to good effect to study the stability of Trojan systems outside of Lagrange's limits and assumptions. This might have applicability in looking for crudely equi-lateral-equi-mass Trojan-ternary star systems currently thought to be "unstable" (see **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**), or looking for Trojan inhabitants of systems thought to be purely binary, etc. If the more general systems could be computer simulated easily, we could get a better idea of how stable/unstable they might be. (One would need to carefully keep track of Poincaré-chaos-type effects in the computations, especially due to limitations of significant real number mantissa bits.) Could they "last" for 500,000 years? or only for 5,000 years? This could be very important in studying the dynamics of stellar nurseries.

It perhaps is still not well appreciated in current astronomy, but the detailed study of Trojan asteroids (along with Trojan "space debris", "atmospheres", if any, etc) could potentially shed light on the evolution of the Solar System in a way that the study of other asteroids (etc) would not. Trying to establish the times at which known asteroids were trapped in orbits around the Trojan points could yield important CSI-timeline clues to the timetable of Solar System development. For example, if even small Trojan asteroids occurred very close to their equilibrium positions with close to equilibrium-zero velocities near the Trojan point(s) in the orbit of Pluto, or even of Jupiter, it would mean that they had probably been equilibrating for a very, very long time (since the atmospheric viscosity of "empty space" is very slight, even if necessarily greater than zero). When we eventually study them up close using space

probes, e.g. studying their composition, the combined information could yield great insights. Ironically, though, the further they are from equilibrium, the easier it might be to estimate how long they had been actually trapped into approaching that equilibrium point. E.g. their Trojan-point-orbital velocities would be more readily measurable, at least with percentage-wise much greater accuracy, and more readily extrapolateable.

Trojan tadpole orbital wells are in unusually precise positions in the volume of Solar System space by astronomical standards, and it should take only a small fraction of the effort to search for asteroids and other interesting things in these orbital wells near planets and moons compared to doing general searches of the whole Solar System. Well, not as precise as one might wish since Trojan asteroids in the orbit of Jupiter seem to range angularly before and behind both Trojan points by ~ 20º, and to range radially ~ 0.5 AU (Astronomical Unit, the average distance between the Earth and the Sun) on either side of Jupiter's orbit. And of course any asteroids in horseshoe orbits e.g. near Jupiter would range almost as far and wide as the more usual sort between the orbits of Jupiter and Mars.

It is important to detect as many Trojan asteroids and determine their trajectories with enough precision to be able to begin to approximate their cumulative effects on each other over time, and in order to factor that into an extrapolated past history timeline. Perhaps it would be practical for our modern space telescopes such as Hubble to be used to search for them, but even if our space telescopes are busier doing other things, amateur astronomers could very likely find both studying known Trojan asteroids and searching for new ones rewarding.

One other fascinating possibility that we started looking at earlier (see **Section 4.3, Trojan Space Debris?! Trojan Atmospheres?!, p. 68**, and **p. 69** for an UPDATE giving a quick description of the still disputed "Kordelewski cloud" near one of the Earth-Moon Trojans) is that there must be micro- and/or mini-asteroids and/or other interesting "space debris"/"atmospheres" slowly accumulating near any system's Trojan points, including the Trojan points of Earth's Moon. This is close enough that we could send a space probe to not only photograph it, but pick up and return samples of such micro-asteroids to Earth with more sterility than picking up meteorites from Earth's surface. And the study of the Moon's tadpole "atmospheres" could be quite interesting. The Moon's L4 and L5 tadpoles might have important differences both in detectable debris and detectable atmospheres, clues to their role in Solar System evolution. (see **p. 69** for an UPDATE on the Japanese Hiten space probe of 1990 which did not detect any noticeable increase in the density of space dust near the Earth-Moon Trojans. See http://en.wikipedia.org/wiki/Kordylewski_cloud and http://www.rfreitas.com/Astro/SearchIcarus1983.htm for all this and more.)

There are various questions about the known Trojan asteroids. One obvious question, one that we have a simple answer to, is why there do not seem to be any asteroids equilibrated precisely at L4 or L5 (here ignoring non-Jupiter Trojans because there are still so few of them, statistically)? One answer, at least partially correct, is that "perturbations" tend to keep them circulating. They can be perturbed by just about anything: planets, other Trojan or non-Trojan asteroids, Jupiter's moons (63+ known as of 2007). But do the known perturbations explain the known orbits within the tadpole orbital wells? Given the known perturbations, is there something like an "equilibrium" orbit that they will tend toward? And if so, are they approaching from the "outside" (having started from a "larger orbit")? or are they approaching it from the "inside" (as if they perhaps started their journey from a "smaller orbit", somewhere "near" the Trojan point proper)?

Another question, still unanswered, is why there so many more Trojan asteroids have been detected in Jupiter's leading L4 tadpole than in its trailing L5 tadpole, or why that particular ratio is found to pertain. (See comment on **p. 63** for a possibility.) And of course anyone who has an interest in hurricanes would want to know how many circulate "counter-clockwise" or "retrograde-clockwise", and if the circulation pattern is the opposite for L4 and L5 (which is "south of the equator"). (These last questions have perhaps already been answered, but not publicly—at least not known to this author—as of this writing in 2007.)

5.4 (More) Trojan Study Possibilities

So we have many possibilities for the astronomical study of Trojan points/bodies in the 21st Century:

- use computers—which Lagrange didn't have—to do more complete analyses of the more general case of 3 non-infinitesimal bodies with no restrictions on relative masses; in particular, look for stabilities that Lagrange may have been unable to find because of his computationally convenient but theoretically limiting assumptions
- try to determine if the orbital data of Trojan asteroids can indicate the time that the asteroids were captured in a given Trojan tadpole/horseshoe orbital well, and study what this might indicate about the evolution of the Solar System
- relatedly, Trojan points and their associated tadpole/horseshoe orbital wells are a good point of focus to study—both theoretically and observationally—the dynamic viscosity/drag of space (from "space debris" and collected "atmospheres", from the tenuous but turbulent larger Solar System atmosphere-wind, both of which will tend to concentrate there if their energies are low enough, etc)
- compared to other projects, it would be relatively easy to send a robotic space probe to pick up and return with micro/mini-asteroids from the almost certainly concentrated space debris in one or both of the Earth-Moon system's Trojan tadpoles, and…
- at the same time make estimates of the amount, distribution and orbits of accumulated space debris, check for the existence of the Trojan tadpole atmosphere, study their composition, viscosity, etc. If both tadpoles can be visited, study the potentially important differences in debris and atmospheres.
- a tangent: why not put solar observatories at the Earth-Moon's L4 *and* L5? There would be less trouble in separating the data signals from the intense solar radiation. They should be easier to keep in stable positions, and easier to maintain in case of equipment failure. And at least one of them would be in position to observe the Sun at all times. This could be combined with other studies of the Trojan orbital wells.

5.5 Simple Approach to Trojans for Students, Amateurs and Professionals

The simple mathematical approach presented here—which is decidedly 2-dimensional and so does not cover the fascinating 3-dimensional dynamics of the orbits of Earth

Trojan asteroids like 3753 Cruithne—can help make beginning study of the fascinating Trojan points in the 21st Century accessible not only to:

- high school physics students
- amateur astronomers (who might be inspired to look for Trojan bodies close to home, e.g. in the Moon's orbit around the Earth or the Earth's orbit around the Sun, and to use computers to study the general Trojan body problem)
- the popular science and popular philosophy of science reading public

but, also importantly, to:

- all professional astronomers and physicists, who (usually) do not wish to spend the extra graduate-level study time and effort it takes to learn the rather arcane and difficult perturbation theory to a useful extent.

And professional astronomers may benefit from this simple approach also, because:

- It is a *simple* approach to certain obvious properties of the dynamics of Trojan points and Trojan asteroids which are not made obvious by the partial differential equation-perturbation theory approach; e.g. many astronomers and physicists are under the misimpression left by Lagrange's approach that the dynamics of Trojan points *must* depend on the 3 bodies having vastly different masses, e.g. a massive Sun, a relatively small 2nd body ($m_2 < \sim 0.04\ m_1$) such as Jupiter in orbit around it, and a 3rd body of asteroid size that is "infinitesimal" in the usual sense of "negligible mass" (which is neither theoretically nor actually realistic if we consider long time periods). But, in fact, at least "unperturbed stability" pertains for 3 *arbitrary* masses, and it may be possible to extend this simple but more general approach to a more general determination of stability (or to inspire the search for such, with the improved understanding that can result even from "fruitless" research). The non-infinitesimality of the 3rd body could conceivably make Lagrange's results inapplicable and/or inaccurate, e.g. it may be that any instability of more equally distributed mass(es) may take an astronomically significant time to show itself, and that astronomers may be able to find more variety in Trojan systems than currently assumed.
- This has implications for e.g. binary star systems and co-orbiting galaxies. The tadpoles of binary star systems can be more closely examined for e.g. proto-stars, planets, or other accumulated "space debris". In the case of co-orbiting galaxies, their "tadpoles" (probably no longer shaped like "tadpoles" because of the distribution of masses) of can be more closely examined for "micro-galaxies", for "condensed nebular",a small particulate or atmospheric matter, for clusters of asteroids, planets or other large masses torn from their previous usual orbits, or, especially in the case of galaxies, for condensed "dark matter". Also, Trojan-ternary star systems may be found to be more stable than Lagrange's theory seems to predict (see **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**).

Newton's Great… Oversight, the difference in falling rates of lighter and heavier bodies, and this related and simple approach to the physics of Trojan points with their

tadpole and horseshoe orbits, just may form the beginnings of a popular and fruitful—and heavenly—body of study for astronomers and physicists in the 21st Century.

6 NEWTON'S—AND SCIENCE'S—GREAT "… OVERSIGHTS"

6.0 Pre-Diatribe…

I have loved science and mathematics—not to mention philosophy, though not the kind that Hippasus failed to "subvive" in his "Poseidon Adventure"—since very early childhood, and because of this, I am an absolute believer in "science… when wrong, to be put right." It would be one thing if it was "only" Newton who had made "Newton's Great… Oversight", but we are in a situation where this "… Oversight" has continued for over 3 centuries, and is continuing still even today, at the beginning of the second decade of the 21st century, both in our science and—far worse—in our science education. Thus this whole chapter will dedicated to encourage cogitation and on and contemplation of this delicately forgivable but stoutly indefensible position that science finds itself in.

For re-starters, let us remind ourselves of the old chess adage mentioned in the **Introduction** (**p. 22**):

> **When a beginner gives away his queen, it's a blunder.**
> **When a grandmaster gives away his queen, it's an… oversight.**

In this chapter and the next we will be engaging in analysis of and heavy philosophizing—of a noticeably polemical nature—about "Newton's Great… Oversight(s)" *and about science in general* from the standpoint of the "Science Wars" that have been heating up yet again in recent decades.

The Science Wars: Many non-scientists, and a gratifying number of scientists, find fault with science on many grounds: moral, intellectual, social, cultural, etc. For example, non-scientists and scientists alike are ever more frequently accusing science of being less "objective" than it pretends to be, finding it, in fact, to be pervaded by subtle but powerful cultural and other biases. Even 21st Century feminists are analyzing science's shortcomings along these lines and bringing additional light to all too "glass darkly" points of view. Though these criticisms are accepted as obviously valid by many scientists, they are stoutly dismissed out of hand by "highly vocal" others as "unscientific".

This complex and complexly evolving situation has in recent decades come to be called the "Science Wars", but we should note that these "Wars" are in fact much older, their evolution dating back beyond Aristotle into unrecorded history, back when "Religion Wars" got *their* start, and probably for much the same reasons. In fact, the "Science Wars" might actually be another co-evolving cycle of the ancient "Religion Wars" hydra. Also in fact, we can think of these "Science Wars", and their "ampolyguously" (more semantically evocative than "ambiguously" or "vague") thesis-antithesis-synthesis-style dynamics as being fundamental to the punctuated evolution of what we have come to call "Modern Science".

These millennia old "Science Wars", in which—anti-thetically—many non-scientists and even many scientists find science to be failing on important but non-scientific terms, but in which—"thetically"—many *other* scientists and non-scientists find science to be "all-but-infallible" in *any* "important" sense, now find a new twist on an old wrinkle:

- Our *modern* science not only *can fail*, but in fact *is failing*, failing in its *own* territory and on its *own* terms, *theoretically* and *scientifically*. And further, our modern science has not only *failed in its own*

> *territory and on its own terms*, but has been doing so—in some cases—*egregiously*, for centuries at a time, continuing up to today.

It is said that "the common man repents of his sins [to which I add almost completely redundantly: pretty much only when they are coming home to roost], but that the wise man always repents first and foremost of his heedlessness." (I first remember reading something like this in an Idries Shah book over 30 years ago, back in the 1970s, but I have read so many that I can't remember which one.) It is way past time for "science"—that's us, actually—to acknowledge, "repent" of and "recant" its—our—own "error(s)" and associated failings, free itself from its—our—own almost religious hubris, but, most wisely, first and foremost repent of its—our—own heedlessness, primarily in general, and secondarily in all the particulars we have observed and/or can guess at so far. We all—but especially we scientists—need to scrutinize science far more closely than we have so far for yet further… oversights, and not just "great" ones.

Everyone who loves science in a

- "Science, right or wrong…"

sort of way should *always* support those essential concomitants:

- "when right, to be kept right;
  when wrong, to be put right."
- It is time for science to, first and foremost—along with "first do no harm"—wisely "repent of its—our—own heedlessness". We need to remember that "truth" is a "thing" of reality, the "territory" that we wish to make "maps" of. We need to stop thinking of our "maps" as "true" or "false"; they are much less than that, and potentially much more than that, if we become and remain heedful. The saying "the price of liberty is eternal vigilance" is a conjoined twin of "you shall know the truth and the truth shall make you free." These are great wisdoms that we need to make the foundation our science, to make "our and our science's rock and our and our science's salvation".

All this is not only important, it is *essential* for the future of our society. As we are urged—even required—to submit more and more to science and its control over our daily lives (or at least to its use as an authority bludgeon—a sort of "New Improved Aristotle"—to enforce such control), it is good to know and remember that even *our* "modern" science is not only fundamentally fallible, as shown by its current failings, but that it can fail in its fundaments for centuries—even millennia—at a stretch without notice, as it did post-Aristotle. It helps to remember that Aristotle was the Newton-Einstein of ancient Greece… and Rome.

And as far as "philosophy of science" goes, it is as John Losee says in the introduction to his *A Historical Introduction to the Philosophy of Science*, p. 1: "Unfortunately, philosophers and scientists are not in agreement on the nature of the philosophy of science." I differ with him, however, regarding the example he proposes of this lack of agreement. I will offer instead two different non-exclusive possibilities:

1) the "philosophy of science" is the general love of general wisdom as applied to the complexly evolving historical-social-intellectual activity that we have recently (in the last 2 centuries) come to think of as "science", but, for example, with *no indication* in this usage that "science" itself is either loved

or itself loves or even contains wisdom (even though these are probably true in some sense),

and/or

2) the "philosophy of science" is the particular love of the particular wisdom that we may perchance find in said "science".

We can Bierceianly note that this of course presumes that there is such a thing as wisdom and that it is loved, or with a sense of hope that eventually there may come to be wisdom among us and that it may come to be loved, in the best senses of those words.

I will proceed with this at times obstreperous analysis and critique of science—lovingly finding much fault with it as well as finding much else that can and should be expected from it—"as if (still in the best sense of the word)…" We can also note that the philosophy of science, along with the history and historiography of science, and with the newly named and still newly evolving field of Science and Technology Studies (aka STS, which previously—way back in the '50s or '60s—stood for "Science, Technology, and Society"), taken together start to form the *beginnings* of a workable "system of checks and balances" which should be considered essential to the proper operation and (punctuated) evolution of science. In science we should also take many inspirations from "Comparative Religion", which was a popular subject when I was an undergrad, back in the… well, a while ago.

- ✓ Comparative Religion is *always* better than "just plain" Religion. We can note that this is why the sacred scriptures remind us that "God is the author of *all* religions." The history of science often starts to be "comparative science", but usually misses the bulls-eye suggested by our already somewhat cultivated sense of "comparative religion", which besides being willing to study religions' successes, is also willing to study their failures, along with everything else. Thinking in terms of "comparative religion" could help greatly in seeing *with new eyes* the history and historiography of science, the philosophy of science, and even science proper, and should be made an explicit source of inspiration in Science and Technology Studies. "Comparative religion for science", yes, that's definitely needed.

6.1 Just How Important *IS* "Newton's Great… Oversight"?!

The Renaissance! In the Bible we can read that "…one day is with the Lord as a thousand years, and a thousand years as one day." We are also told in the Bible that the time from when Jesus died and descended to the dead till the time He was resurrected was (approximately) a day and a half. That day and a half works out to (approximately) 1500 years, the (approximate) time the Renaissance commenced. It cannot be overstated that one of the most essential insights into the character of the Renaissance is that its Great Light (obviously corpuscular, but with a temporary hint of undulatory) Newton made this "Great… Oversight".

Newton can be considered a "grandmaster" of natural philosophy, and even of the "modern science" that the natural philosophy of his day has since punctuatedly evolved into. Therefore, his failure to notice that his own theory predicted that lighter and heavier bodies (generally) fall at different rates can be satirically graced with the term "… oversight". This… oversight (by definition unnoticed at the time) certainly would have been, and we can say was, important enough scientifically—naturally-philosophically—in its day. Newton should *never in the world* have failed to apply his own new "law of gravity" to the same-falling-rate finding of Galileo. Newton might literally have died of embarrassment if one of his lesser contemporaries, or even

Leibniz who was rather more his equal, had made public the derivation-analysis given above of Lagrangian points L4 and L5, even without a simple proof of stability.

- There is one very good measure of how great "Newton's Great… Oversight" is, and that is to imagine Newton's reaction if his "loathed enemy", his Moriarty, Robert Hooke, had presented it before the assembled membership of the Royal Society. Even God might have trembled at the thought of Newton's wrath: "You… *You* let that… that *creature* of Yours do *that* to *Me?!*"

Newton's contemporaries, a few "grandmasters", somewhat more "masters", and lots of beginners, should *never* have so failed either. It was, and still is, very important historically, philosophically and psychologically—and somewhat terrifying overall—that they *all* so failed, that *we* all so failed, and are still failing even as I write this.

If the reader objects to "terrifying", remember that we are speaking of Isaac Newton himself, by most accounts the greatest scientist who ever lived and the discoverer of the "Law of Gravity" in question, as well as speaking of his contemporaries, some more noteworthy than others. *He*, at least, should *never* have overlooked the non-zero falling rate difference, not even for an instant, not after that Apocryphal Apple Fell on his Mythical Head. And, extremely important and even more terrifying, this great… oversight has continued for over 300 years, even up till now, the end of the first decade of the 21st Century.

Another, far more grave source of terror that we all should feel is… *anger*, anger such as the modern scientists of Galileo's day—empowered by, as "well" as empowering, the Inquisition—felt at Galileo's questioning of Aristotle, anger such as Newton himself felt at those who had the temerity to (even seem to) call into question Newton's "corpuscular" theory of light by proposing the then seemingly contradictory "wave" theory possibilities. Another clear example of this is the literally angry treatment Thomas Young received at the hands of "Newtonian Inquisitors" in the early 1800s when he tried to publish his work which supported the wave theory of light. (See I. Bernard Cohen's Preface to the revised Dover edition of 1979 of Newton's *Opticks*, p. xi.)

And this is not a merely historical, pre-modern science phenomenon. In the 1990s a leading scientist (here traditionally nameless) literally became overtly *angry* at my Internet questioning of established scientific belief, especially of such a basic belief as the equal falling rate finding of Galileo, when the case for the difference in falling rates was first presented to him. Only grudgingly did he *later* agree to actually look at the equations and analyses. Only grudgingly did he then finally admit that the falling rate difference did actually exist, but he simultaneously disparaged the result, saying roughly "it's too small to be scientifically important." When it was pointed out to him that physics is now *proud* of detecting the "infinitesimal" advance in the perihelion of the orbit of Mercury and its tremendous scientific importance… all further (e-mail) conversation came to an abrupt halt.

- "Scientific" anger at questioning established scientific belief is far more terrifying than even great scientific… oversights. Newton's anger at his light-wave "opponents" (especially Robert Hooke, remember) can be said to have effected the re-establishment of this ancient Religious… "precedent" in *Modern* Science, where it holds power—the power of the Inquisition—over our scientific lives even to this day. The freedom—*our* freedom— to find fault with science is *fundamental* to true science, *never* a danger or threat to it. Even if you don't look through the proffered telescope or microscope, anger at "scientific counter- revolution" or even "pseudo-science" has *no place*

whatsoever in true science. We should always keep in mind that much "modern science" was once "pseudo-science", e.g. the wave theory of light.

It is extremely unlikely that anyone will say that the discovery of this falling rate difference… oversight of Newton—and most unfortunately also of every scientist since Newton into the beginning of the 21st Century—terrifying though it is, is… *The Apocalypse*. The wave concept of light—when *properly* married to the particle concept, espoused "corpuscularly" by Newton—led to quantum mechanics. The "infinitesimal" advance in the perihelion of the orbit of Mercury might spell the difference between Newton and Einstein. But the falling-rate-difference question seems to be just… Newton, and a signal affirmation of his theory of gravity, at that.

- ✓ NOTE: the Earth is not actually a Lorentz frame, i.e. it is not an inertial frame of reference in "uniform motion" relative to an absolute Newtonian-style frame of reference, so we cannot expect bodies with different masses to accelerate—as observed from our *Earth's* frame of reference—at precisely the same rate as relativity holds that they do. This entire situation is fascinating from a historian's perspective, demanding from a psychologist's perspective, but distinctly terrifying from the perspective of a true lover-philosopher of science… or truth.
- ✓ FURTHER NOTE: the concept of a Lorentz—or inertial—frame of reference is purely a gedanken concept *IF* we need such a frame pinned to any bit of matter-energy, such as the Earth. (Conceivably there could be people in "other dimensions" watching us without our limitations relating e.g. to Heisenberg's uncertainty principle and other such things.) Any bit of matter-energy is constantly accelerating due to the "countless" forces from gravity, electro-magnetism, etc. Due to the relativistic equivalence of matter and energy, even a photon (or a neutrino or a quark, which are more obviously "matter") cannot in reality be—or be the home base of—a Lorentz/inertial frame.

But, thought provoking as it should be, the non-zero falling rate difference is unlikely to provoke any revolution or shift of *standard* scientific paradigm, in the sense of early Kuhn. (See *The Structure of Scientific Revolutions*, 1st edition. Kuhn—I think unfortunately—"recants" somewhat in his Postscript of the later editions, so check those out, too.)

Actually, that last needs qualification. Realizing that, in scientific theory as well as scientific fact, lighter and heavier bodies fall at different rates will not really improve e.g. today's rocket science. The falling rate differences would not be critical there, and would largely be taken into account implicitly in the equations and calculations that the computers carry out (e.g. on rocket, Moon, and Earth). We already knew about Trojan asteroids, by way of Lagrange, and about the possibility of putting space stations at Trojan points.

- ✓ By the way, putting a SOHO type solar observation satellite at an Earth Trojan is a good idea, even though it would be almost 100 times further away from the Earth than SOHO. The data signal back to Earth wouldn't need to compete with the Sun's… uhh, signals, at least not the way SOHO does. It wouldn't need all that fuel to maintain its position (using the figure-8 loop). And it could look for other things, like tiny nearby Trojans, too… and tiny Greeks! The same goes even further for putting SOHO type

observatories in the Earth-Moon's L4 and L5 points. They would be much easier to place there and even to service in case of "technical difficulties".

Astronomy will be affected rather more, because there is a good chance that this result will yield inspiration to reexamine the Lagrangian concept of Trojan points and bodies and their "stability". After all, as of November 2011, astronomers still think that horseshoe orbits and tadpole orbits are separate phenomena! And of course it will make Trojan astronomy more accessible to all, especially us ordinary mortals. New possibilities will be looked for and at, such as quasi-equi-mass Trojan-ternary star systems, currently thought to be unstable per Lagrange's analysis. (See also **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**.) But, still, this does not constitute a "standard" paradigm shift (if there *is* such a thing at present in science).

So "Newton's Great… Oversight", as we have been exploring it in this ebook, will not bring about even a hint of a paradigm shift away from Newton's theory of gravity, rather the opposite: it didn't fail us; we failed it. It is yet another (non-relativistic) affirmation of the utility of Newton's theory.

- ✓ Oops! I should have made it clear before that we fail Newtonian theory every time we wrongly speak of the parabolic arcs of cannonballs instead of their sections of ellipses, which to be sure are still only abstract gedanken trajectories.

One possible minor paradigm shift, though, lies *within* Newtonian theory. With the example of this almost miraculously simple mathematical approach to Lagrange's Trojan asteroids that largely avoids Lagrange's arcane perturbation theory, many astronomers and physicists may very well start to look for similar super-simple mathematical approaches to problems that have so far proved intractable because we have not had a sufficient supply of Lagranges to give us yet more arcane solution methods.

The real paradigm shift, however, will be a "meta-paradigm shift", a conjoining of the "Science Wars" with Kuhnian "Scientific Revolutions" (which should probably be better understood as "Punctuated Scientific Evolutions") This "meta-paradigm shift" will be a true apocalypse for some. At least a start of this "meta-paradigm shift" will be needed to address the all but impossible task of explaining *just how*, not only *Newton himself*, but *every scientist and science educator since* has "… oversighted" that Newton's 300++ year old theory itself predicts with miraculous simplicity not only that lighter and heavier masses will generally fall at different rates, but that their gravitational interactions give rise to Lagrange's L4 and L5 and their associated Trojan asteroids in their tadpole and horseshoe orbits. The simplicity seems even more miraculous when compared to Lagrange's arcane perturbation theory, also fundamentally Newtonian.

We must raise the question of to what extent and how this "… oversight" *and* its continuation for 300++ years is an (avoiding the harsher terms) "… oversight" of scientists and/or of science and our practice of it, of our Kuhnian "normal science". "Repenting of our heedlessness" must be made an essential part of "… when wrong, to be put right." There is a very large "wake up call" being sounded here, and it is my hope that "Newton's Great… Oversight" will be an impossible to ignore harbinger and herald of that call.

It is to be hoped that studying "Newton's Great… Oversight" will ultimately inspire appropriate change in some of the directions of the *punctuated evolution* of our *philosophical* and "*psychological*" paradigms, change that will produce better foundations and better meta-foundations for all future scientific endeavor, the "meta-

scientific methods" from the implicit evolution of which we unconsciously co-evolve our "Scientific Methods"—accent on the plural. The use of the turn of phrase "the punctuated evolution of scientific methods" is quite apt, since we historically tend to think both of "The Scientific Method" as being singular and as being "The Pinnacle of (such) Evolution", "the Survival of the Fittest" (always a bogus concept: only the Olympic gold-medal winner lives on… rubbish!) In fact we have evolved almost as many "forms of scientific method" in our "modern science" as Mother Nature has evolved "forms of life" in nature. (That last, shall we say cutely, is numerically an exaggeration, but quantitatively and qualitatively right on the money.)

Our modern "Science Wars" have never—at least, not up until now—taken science to task for failing on science's *own* terms, in its *own* territory. Here, however, we will start to take a (quick) look at the systemic and systematic nature of at least this one scientific error and scientific failure in science.

6.2 Newton's Great… Oversight: How?! Why?!

Actually, since it has to do with Newton's character, we will stray somewhat from the intended seriousness of this analysis to somewhat humorously remind the reader of Newton's feelings towards Robert Hooke. Newton loathed Hooke, and one of Newton's best known sayings, which makes him seem humble when taken out of context, cleverly but unworthily casts aspersions on Hooke's "natural philosophical" competence. The full import only makes sense, though, if the reader further knows that Hooke was of very short stature, in large part due to *extremely* seriously deforming childhood health problems. Thus we get (a variant of one of the two most popular flavors of) Newton's famous saying:

- "If I have seen further than others [alternatively "than you and Descartes"] it is by standing upon the shoulders of Giants."

  Newton, in a letter to Robert Hooke, dated February 5, 1675
  (or maybe 1676. In any case a remark worthy of neither of them.)

We can just hear Newton whispering: "so we know whose shoulders I have *not* been standing upon to see further than others, especially further than *you*, *don't* we, *little man!*" So we can one-up by cleverly noting that perhaps Newton committed his Great "… *Oversights*" because while he was busy "seeing further", he was also busy failing to see past the end of his own nose.

Now we get back to not just a serious, but an essential question: How could such a simple, obvious result—the Newton-theoretically predicted and physically real non-zero falling rate difference of lighter and heavier bodies and its essential relation to Lagrange's Trojan points—be overlooked by "the Genius who discovered gravity"? by "the Lion"? (or "his Paw"?) by Newton himself?!

The answer must lie partly in Newton's character, of course. (For this the reader is referred to other sources concerning Newton's character, such as Westfall's magnum opus Newton bio, *Never At Rest*.) But here we will look rather more at the character of the Renaissance that was budding-flowering during Newton's life, a "rebirthing"—literally a "resurrection"—that went far *far* beyond even the immense changes taking place in "natural philosophy", changes that spanned millennia in the West, and even more millennia in the East. We will forgo descriptions of Newton's genius and character, and of his role in this rebirth, to focus on what was happening spiritually and intellectually in his day.

Europe was beginning to throw off the suffocating rule and the stultifying authority and power of Religion, as it had come to be practiced in the Western World. Galileo

had actually been in peril of his life from the Inquisition, and many others had lost their lives—and, worse, their souls—to it. Aristotle—ironically, a pagan Greek—and his teachings had come to solidly represent, and underpin, this ostensibly religious rule, this ostensibly spiritual authority and power.

The world was not then known for great wisdom, nor is it today. Because Aristotle and his science represented the Religious rule-authority-power that many were trying to liberate themselves from, including the budding European neo-intellectual and neo-natural-philosophy-evolving-into-science communities, these many found it "necessary" not just to *find fault* with Aristotle, but to *dismiss* him, even to overtly *depose* him and his teachings (but not so quickly as to get into *too* much trouble). Note a classic recurring theme: Aristotle was by this time really just a *club* used by Powers-That-Were (and still That-Are to a great extent) to beat and/or threaten people into submission. (Calling this an "Aristotelian Inquisition" would be an exaggeration, but an insightful one, as mentioned toward the end of the **PRE-SCRIPT, p. 15**.) But it was still too soon to defend oneself directly against the *clubber*, so… *"attack the club!"*

The work of Galileo concerning falling bodies flew in the face of *some* of Aristotle's teachings, overthrew them, in fact, all too publicly helping to hasten the waning of the supremacy of the Church and Its Inquisition, that Mystical Marriage of the Anti-Christ with the Christ, which had long since dedicated itself to taking "all rule and all authority and power" unto itself, as much as the traffic would bear. It was far too soon to think of questioning this new Golden Calf, or perhaps "this new Moses", Galileo and his Two New Sciences (one on each Tablet), which had helped drive out that Sacred Cow, or rather Sacred Bull, symbolized by Aristotle and his now partly outmoded philosophy-science.

If Newton or his contemporaries had however mildly contradicted *Galileo*, *even on this one point*, even with overpowering equations and other rationalizations, to these newly self-established Renaissance Renegades, Newton himself among them, it would have been tantamount to turning traitor to their newfound salvation, traitor to their divinely inspired but still importantly self-achieved liberation from ignorance and error, traitor to this new life that was beginning to form in-and-as this Brave New World, life that they too were parentally re-conceiving and giving re-birth to, and which quickening life they themselves *were*, as well. To even mentally form any seriously critical questioning of the by then Sainted and Apostled Galileo and his miraculous new Dogmas would have been the Renaissance equivalent of questioning Divine Providence: Not Allowed! even when trying to better know the Divine Essence and/or Divine Will through studying Nature. These Renaissance Renegade natural philosophers could no more have said no to their Galileo than their pre-Renaissance pre-Renegade predecessors could have said no to their… Aristotle.

But… there is still the crucial and excruciating question of why *no* scientists *since* Newton have publicly questioned the no-falling-rate-difference finding-hypothesis of Galileo. Here we will merely note that, if by the time of Newton Galileo had become a neo-intellectual-natural-philosophy Apostle, almost a demi-god, Newton himself—even today "generally recognized as the greatest scientist who ever lived"—soon became all but God Himself, almost within his lifetime.

- **"Nature and Nature's laws lay hid in Night:**
  God said, Let Newton be, and all was Light"
  Alexander Pope (1688-1744)

If "***He***" didn't question Galileo…

6.3 Comparing Newton's Three Great… Oversights

We need to reiterate that Newton made at least three "… oversights"—one mathematical and two scientific—that can be considered great:

1. Newton overlooked Leibniz's simple and insight inspiring $\mathrm{d}f(x)/\mathrm{d}x$ notation for his calculus;
2. Newton overlooked the wave nature of light (though, ironically, he at first ascribed to it an "undulatory nature", later backing off since we can't be expected to imagine him actually "recanting");

and, as we have been studying here:

3. Newton overlooked that his own theory of gravity predicts that lighter and heavier bodies generally fall at different rates.

It is natural, and quite useful, to compare Newton's two most notable scientific… oversights (the second and third, above): the wave nature of light, and the non-zero falling rate difference of lighter and heavier bodies. These oversights are quite different in a number of ways, and both are crucially important regarding the history and current state of modern science and its philosophy. A serious comparison of these oversights is worthy of a book or three, so we will merely dabble here. In the next section we will try to use his *relatively* well-known (but still mostly "overlooked") oversight of its wave nature to shed more light on Newton's heretofore *completely oversighted* oversight of the falling rate difference.

6.4 Newton's Great Wave Theory "… Oversight" and the "Science Wars"

As mentioned in the **Introduction** (**p. 23**), even today it seems incomprehensible that Newton, "the greatest scientist who ever lived", completely rejected any "wave" concept of light in favor of his own "corpuscular" theory of light (which had significant differences from our modern "particle" concept). Actually, Newton's theory was not strictly "corpuscular", although it is usually remembered as such. It was actually more general, even extending to allowing light to have an "undulatory" nature, but not a "wave" nature. (The whole story gets very complicated. In some ways Newton seemed early on to be anticipating the wave-particle duality we now find in our quantum mechanics, but he later both played down his "undulatory" forays and distinctly *denied* light *as* waves—maybe that ol' alchemical mercury acting up, again. See Newton's *Opticks*, especially the Preface by I. Bernard Cohen and Introduction by W. T. Whittaker in the revised Dover edition, 1979. Or maybe it was because he associated the wave theory possibility with the Hated Hooke.)

Even more incomprehensible scientifically is that Newton more or less deliberately (even if founded in profound petulance) used his preeminent position in European "natural philosophy" (as science was known at that time), in the academia of that time and in European society in general, rather than pure "science"—well, "natural philosophy"—per se, to "scientifically" quash the "light as waves" concept, which he probably even *thought* of as crushing the "rebellion" of his wave-oriented scientific "enemies"—for such is how he seemed to think of them (at the very least Hooke, who had earlier publicly "shamed" him before the Royal Society, and probably the others as "guilty by association"), at least under the influence of all the mercury he consumed in his secretive explorations of alchemy. He was so "successful" in having the concept of light as waves made "scientific heresy", at least by his "natural philosophy heirs"—even if he only subconsciously held that intent—that a wave concept of Light would have to wait through almost two centuries of "Dark Ages" from the publication of the

Principia before light-as-waves was finally and grudgingly somewhat accepted by "science"—which had only just changed its name away from "natural philosophy" in the 1830s—eclipsing Newton's "corpuscular theory", mostly through the efforts of the all too short-lived James Clerk Maxwell (1831-1879). Yes, yes, Thomas Young (a prominent victim of the Newtonian Inquisition) and his interference experiments in the very early 1800s helped… and of course Fresnel… but by the 19th Century "theory" with its "mathematics" was co-opting leadership in "science" from "experiment" (which had really only started blooming in the 16th Century), Michael Faraday to the contrary notwithstanding.

Mild digression: Again, see Miles and Peters II's "Seeing Further, The Legacy of Robert Hooke" at http://starryskies.com/articles/spec/hooks.html which—if I am reading it right—suggests the psychologically interesting idea that Newton rejected the wave theory of light *because* of his first run-in with his Moriarty, Hooke. In 1672, Hooke, who had already recently published his wave theory of light, publicly "rebuffed" (I like *"questioned"* slightly better) Newton's first paper on light, which he had just presented to the Royal Society. "Speaking volumes" about Newton, they write simply (quoted here without permission, as of this writing):

> "… [Hooke] claimed Newton had not proven his idea clearly, and needed more detail. … Newton had the equivalent of a temper tantrum. The situation was made worse for Newton because Hooke was not the only one attacking Newton's theory, he had been joined by Christian Huygens, Ignace Pardies and the Jesuits of Liege. Newton had, since childhood, reacted strongly to criticism. He constantly challenged authority, and to rebuff him, was to become an enemy. Newton demonstrated this over and over during his lifetime; his response was often either complete withdrawal, or open battle. On this occasion, Newton chose withdrawal (though usually for Newton withdrawal was some form of manipulation in battle plans.) In March 1673, Newton wrote to Henry Oldenburg, the current secretary of the Royal Society. Newton requested to withdraw from the Society. It took much gushing of admiration, respect, etc. on Oldenburg's part, as well as an offer to wave dues to the Society to get Newton to change his mind. Oldenburg also offered an apology for the behavior of an 'unnamed member.' The stage was set. Newton had successfully established his place in the Society, and had scored a victory, of sorts, over Hooke."

It seems to be generally allowed that Newton, for pretty much his whole life, never accepted anything even faintly resembling criticism well, and anyone who held an "opposing opinion"—even the widely supported "wave theory of light"—he counted, not just as a critic, but as an actual opponent, or worse. Later in life he only allowed people around him who were variants of pseudo-Renaissance "yes men". Westfall, on pp. 801-2 of his gratifyingly thorough Newton bio, *Never at Rest*, gives the example that just a couple of years before his death, Newton was insisting that a certain comet had a *parabolic* orbit, when his theory clearly says it should be elliptic. Halley, ever a supporter of Newton, wrote to Newton slavishly apologizing for he, Halley (rather than Newton himself), having made the mistake in previous calculations, and hoping that Newton would forgive him and publish the corrected data for the comet. Newton ignored him and refused to have the corrected comet data published with the rest of the work that was printed soon after. That obviously wasn't all mercury or senility, n'est-ce pas?

In any case, Newton's "followers"—for centuries after—tended to have Newton's imperial despotism in their makeup, and they treated anyone who dared to be the least

critical of Newton or his work, however faintly, however implicitly, with publicly expressed anger, contempt and "natural philosophical" dismissal. The classic example of their rejection of "further work" in any of the areas that Newton had worked in and persecution of the "further workers" was the case with Thomas Young, who was performing interference experiments in the very early 1800s that showed the wave nature of light and trying abjectly to appease his "Newtonian Inquisitors" by profusely giving Newton credit as having established the starting point for these researches, which seemed to contradict the "corpuscular nature" of light that Newton had espoused, even though it was in accord with the "undulatory nature" of light that Newton had waffled on over the years.

Enough digression.

However, soon after "The Revenge/Return of the Wave-Theorists", Newton's "corpuscular theory" regained a restrained "co-equality" as quantum theory evolved, and now neither seems to be "more equal" than the other ("down on the Farm"). Their marital union is now known as the "wave-particle duality of light", and glorified in the now traditional scientific paradigm(s) of our quantum mechanics. The scientific world—not to mention our educational system—"conveniently" (and "benignly", even if in the worst sense of the word) "neglects" Sir Isaac's role in the punctuated scientific evolution(s) that has (have) brought the "wave-particle dogma" to its 15 minutes of fame at the top of the heap, the new wave paradoxically but inspiringly sharing that heap with its old rival, the "corpuscle, now newly renamed—and importantly reconceived—as the "particle".

This Great Wave "… Oversight" in particular of Newton is both "better" and "worse" than the Falling Rate "… Oversight" that is the subject of this book. The "… oversight" per se is "better" because it is *somewhat* more justifiable psychologically and scientifically (but… *Newton?!*) It is "worse", however, because of the implicit "Newtonian Inquisition" and its quashing of the "wave theory of light" that put the physics of Light in a "Dark Ages" for almost two centuries after his *Principia*. (See I. Bernard Cohen's Preface to the revised 1979 Dover edition of Newton's *Opticks*, p. xi, for a clear example.)

"Science Warriors" and other participants and/or observers in the Science Wars should pay particular attention, not only to Newton's Great… Oversights (we have mentioned three, and we should expect to eventually discover more, besides the business about 8 decimal places of accuracy, even though it begs to be considered), but perhaps even more to Newton's horrifyingly *ab*-using his preeminent position to crush his "scientific enemies". This is the kind of suppression usually only associated with the Inquisition—most famously the "Spanish Inquisition", but of which there have been and still are many varieties, species, genuses…—and other suchlike "powers". (The reader is encouraged to read 1st Corinthians, 15:24, for a Biblical comment on such that is even more important today than it was two millennia ago.)

In one variant or another, the power of the Inquisition continues even today, even in the highest offices, not just within our religions, but within our cultures, societies and socio-political-religious organizations, and, unfortunately, even within science, from which many have long hoped for much better. Newton, after all, was one of the great lights in the Renaissance wave of "Science Wars", which we can also term "Inquisition Wars", in which the newly renascent, continuingly renascending, world was still struggling to attain salvation from what seemed to them—quite rightly—to be the rule-authority-power of the prophesied "Anti-Christ" that made itself manifest in part as "The Inquisition"—at the very least.

Quantum mechanics could have been discovered much, much earlier, were it not for this "Newtonian Inquisition", in poignantly ironic counterpoint to the famous "Spanish Inquisition" (remember Isabella?… and Ferdinand?), which latter may still have been the very Flavor-of-The-Century Inquisition that threatened the life of Galileo. Serious suppression of this overt sort, as terrifying as it is, is not merely historical. It still happens quietly, day-to-day in our modern world, *even within science*, which once led so many to hold so much hope to so many for freeing us forever from all such suppression. The advent of natural philosophy and science overall may have helped free us from much error and ignorance, but the new outbreak in recent decades of the age old "Science Wars" warns us yet again that science too can fall prey to the lure of "all rule and all authority and power"—which sacred scripture warns us *all* to eschew absolutely, along with judging and punishing—as it did so cripplingly for natural philosophy as it punctuatedly evolved into science (and for our freedom-salvation in general) in those heady anti-wave days of the "Newtonian Inquisition", which, as with religion before it, they self-justified by their self-righteous self-perception that they were finally putting serious "untruth" out of the picture.

- ✓ Reminder: "Newtonian Inquisition" is not an expression you are likely to see or hear often any time soon, since—as the late Thomas Kuhn of *The Structure of Scientific Revolutions* fame would likely have agreed—Science (like Religion and many others), when it tells its story, when it writes its history, does not like to remember and repeat the parts of that history that involve it's Heroes—and their priests, acolytes, devotees, and sycophants—having "innocent blood on their hands".

Even by the standards of today's science, Newton was justified in evaluating competing hypotheses for the "nature of light" and "naturally philosophically" rejecting one of them when it seemed the lesser of the two in explanatory power or other natural philosophical goodness. We can *now* see that Newton was scientifically wrong—and egregiously so—in his scientific rejection of the "wave nature of light", and "even wronger" in his suppression of his wave hypothesis "enemies". We can also now *begin* to see that our modern science is wrong (in general) in such scientific rejections (in general), thanks to our current hindsight-advantaged quantum mechanical understanding that light has *both* a "wave nature" and a "(somewhat Newtonian) corpuscular nature" or "particle nature", light's "wave-particle duality".

If Newton had even merely countenanced the competing wave hypothesis of light, it is likely that a non-mathematically based quantum mechanics would have been "strongly experimentally suggested" by Faraday and theoretically developed by Maxwell and Heaviside, or perhaps even someone(s) earlier. In fact, if Newton had *championed* the hyper-spatial-temporal concept of (competing or other) hypotheses, he *himself* might have discovered-founded a quantum mechanics.

- ➢ This de rigueur standard elimination of "competing" hypotheses is actually a fatal flaw in our modern Scientific Method. This will be discussed in more detail in the next chapter.

It may seem like kicking someone when they are "down", but here is as good a place as any to question science's vaunted—we should acknowledge it as arrogantly vaunted—"scientific objectivity":

- ➢ *Just where* is the "scientific objectivity" in "Newton's Great… Oversight" (any of them, for that matter), and *especially* in its continuation by *all* natural-philosopher-scientists and educators for the last 300++ years?!

And to try to one-up Bertrand Russell:

- Have no respect for the objectivity of others, for there are always contrary objectivities to be found.

And, to try to one-up Arthur C. Clarke and Larry Niven, a future insight:

- Any sufficiently advanced-evolved objectivity is indistinguishable from subjectivity.

And, perhaps merely to exercise our terminal cuteness, we can note:

- On being "Completely Objective": there are an infinite number of possible Objectivities, and choosing one is a Completely Subjective process.

Which may lead us eventually to T. S. Elliot's:

- We shall not cease from exploration
  And the end of all our exploring
  Will be to arrive where we started
  And know the place for the first time.
  T. S. Elliot, *Four Quartets*, *Little Gidding* (the 4th quartet)

A bit oxymoronic, but inspiringly so!

6.5 Questioning Scientific Dogma

There is obviously no such thing as "Scientific Dogma" because Science doesn't have Dogma. Only Religion has Dogma. Science has… uhh… "Scientific Truth", and the "Scientific Method" that makes it inevitable. And, terrifyingly, we are being taught more and more every day that there is no other kind of truth and method, or rather that there should not be.

Less satirically, we are made aware yet again and ever more frequently of the stultifying effects of insistence on *not truly critically questioning* "beliefs", scientific or otherwise, even of covert such insistence. Science—like religion—has accepted too much on the wrong kind of "faith". It is well to remember: the *original* (Biblical) meaning of "faith" and "belief" in religion was not what it is today; it was *never* any kind of willful ignorance—"we won't look through *your* telescope, because *we* have… *faith!*" It was—and still is—rather an observation-communication-connection, a "let them who have eyes see; let them who have ears hear" kind of thing, and "you will know the truth of the *whole* of reality, and the *whole* truth will make you *wholly* free", from which we have sadly departed. For the ancients, "to know" was an active ongoing sense-perception process, including senses and perceptive abilities not yet awakened within us or even evolved to the extent that they could be awakened. It was not a mental-intellectual equivalent of the "book learnin'" that from time to time falls into various and well-deserved ill reputes.

The theoretical falling rate difference was readily accessible to Newton's contemporaries through his theory of gravity (at least post 1687 publication), not to mention to Newton himself, and it is incredible that not only did they all miss it, but every astronomer and physicist since has missed it, as well. Perhaps we should note again that the temper of Newton's times was one which was still not intellectually or spiritually free of the repressive affects associated with Aristotle's… eccentricities, at least not free of the uses to which they were put by those in positions of rule-authority-power. Even after Copernicus these remained quite powerful, even as they slowly waned. Today, the Inquisition—renamed-repackaged-remarketed-remerchandized, to be sure—still finds itself in rule-authority-power in the highest offices in "this system of things". Tomorrow…

Guilt by association, innocence by association, even *association* by association, are all examples of popular and all too often extremely deadly fallacies. There was an overtly felt need to reject *all* of Aristotle, even when he may have been right, since he was both a symbol of the spiritual *and* earthly rule-authority-power of e.g. the Church, and a heavy club to beat down "heresies" and "heretics" who didn't want to worship, serve, or submit to that rule-authority-power. The struggle was very much one of the young Turks of scientific THEORY versus the old Greeks of religious THEOLOGY. We can note with serious irony that the "THEO" in each is (from) the *Greek* word for "God", meaning that in essence "THEORY and THEOLOGY are actually spelled the same". And both of these tend to lead to internecine—and even intranecine—warfare. The War of the Roses may be over, but this "WAR OF THE THEOSES" continues even today. A Sacred Cow may sometimes be attacked in a Politically Correct manner, but *only* if it is on the *wrong* side of the fence, or has come *from* the *wrong* side of the tracks, or can otherwise be shown to be deficient in proper pedigree.

Galileo was not only an Apostle or Saint to the new Renaissance intellectuals, for standing up to the Inquisition (even if he was ultimately moved to stand down), he was on Newton's side of the fence—and today's modern science's side, the right and proper side of the fence—and therefore to be devotedly and passionately "preserved, protected and defended" from truly critical questioning, "preserved, protected and defended" from "when wrong, to be put right" if not from "when right to be kept right". Galileo, who had set out to *overthrow* the old dogma, had himself *become* the *new* dogma; and Newton could not bring himself to be other than Galileo's "Champion", Newton and everyone since.

- **We should *all* be asking how Newton could have overlooked that his own theory predicts that lighter and heavier bodies fall at different rates, and in such a spectacularly fascinating fashion.**

  We should also *all* be asking how every scientist and science educator since Newton has also failed to notice this, and we should be ready to be terrified at the answers, as there are surely many more than just one "simple" answer.

6.6 Our Aristotles, Our Galileos

Galileo is hard to fault since he had no real chance of apocryphally measuring a falling rate difference of $\sim 5 \cdot 10^{-18}$ m/sec^2 let alone of $\sim 2 \cdot 10^{-24}$ m/sec^2 from the top of the Tower of Pisa using his pulse or even a mechanical clock, and since he as yet had no *theory* from which to derive it, no *metaphysics* to base such a guess on. (I will comment further on metaphysics in science in the next chapter.) Newton, however… and every physicist and astronomer since, at least into the early 21st Century… well, because science is really still in its infancy, or more insightfully is really still being *conceived*, perhaps we should continue to grace the situation with the term "… oversight". "Blunder" sounds so "… harsh", even if more than justified. If there are any "scientists", however, who would dare to maintain anything like "the difference is so small, it's scientifically negligible!", we should negligibly quote Galileo:

- "Eppur si muove…"
  "And yet it moves…"
  Galileo Galilei

Relatedly, Aristotle has historically come through this falling rate difference contretemps not all that well. His falling rate thesis… well… it is mainly humorous to point out that he was 2/3 right. Perhaps we should allow that Aristotle, too, did not blunder, that his, too, was an "… oversight". And when we stop to think that we are *still* teaching in our schools a 300++ year old… oversight, it is good to remember that, of all people, Aristotle also said:

- "The fate of an Empire depends on the education of its youth."
  Aristotle

So, perhaps Aristotle doesn't come out looking so bad. After all, we really *can't* afford to go on teaching science's "… oversights" as "truth", not even as "scientific truth", nor can we afford to go on teaching our future scientists to be angry when people point out and critically question… The Emperor's New Fall Lines. And Aristotle *was* 2/3 correct (sort of)!

Here at the beginning of this new ("three is a charm…") millennium, as we try to start to try to put it right, let us hope that science will soon give up trying to be the new religion, with its—now tacitly-implicitly promoted, as well as more humble—position of "all-but-omni-*science*" and "all-but-infallibility" (both of these, however, still in the "objective" "practical" senses, e.g. the senses that "justify" the *properly credentialed* orthodox rule-authority-power to impose "order" on people's lives), and its more explicitly public stance of science-versus-the-"subjective"-world that can be poetically rendered as:

- "If it ain't dreamt of in *our* philosophy (there's no need for more than one),
  it just ain't!"

Questioning *Sacred* Cows is still considered heresy—even if we use other words for it—in today's world, even in today's world of science, so we can guess how the scientists of Newton's time must have felt, both as old-regime heretics themselves, *and* simultaneously as the new establishment, beginning dishearteningly quickly to "Inquire" into the beliefs of new-regime heretics. The Inquisition has won to this extent, that those who later came to truly question the new Scientific Dogma did so in a strictly imposed silence, and all too often still do.

[This page intentionally left blank.]

7 TOWARD NEW SCIENCE AND NEW PHILOSOPHY OF SCIENCE

7.1 "… Oversights" and the Dynamics of Science's "Punctuated Evolutions"

One of the formidable problems we face with "scientific… oversights" is that they themselves can all too easily become and/or form the basis of *paradigms* in the sense of early Kuhn. These "… oversight" paradigms can be held onto as fiercely as if "precious" (by Gollum). Newton, remember, fiercely resisted both Leibniz's superior calculus notation and even a temporary ceasefire with his "light as waves enemies", starting with Hooke. In fact, for unfathomable, esoteric, spiritual-psychological reasons, "error" is often far more "precious" to us than… uh… "non-error". We never really "detach" ourselves from our "original sin", preferring to "cling" to it as if it were "our rock and our salvation". This has been true for ages in religion, for example, having evolved long before "natural philosophy" first saw the "light of day" and started to take its first faltering breaths.

It is good to remember the work of Kübler-Ross. Most scientists or academics who are confronted with any significant indication of scientific or academic "error" will—almost certainly—initially react in terms of variants of the first two Kübler-Ross stages: *denial* combined in some amplitude-ratio or other with *anger* (as was mentioned in some depth earlier). One also often sees signs of that other famous duo, "fight or flight". One can all too easily find such responses when attempting to publish in a scientific journal.

The denial part was an explicit issue for Thomas Kuhn in his *SSR*, the denial paradigmatically lasting until a generation had passed (in both usual senses), until the deniers belonging to the Old Greek Guard had passed away, leaving the now "mature" scientific revolutionaries as the Young Turk Guard. But for some reason or other Kuhn decided not to make an explicit issue of the *anger* part. One can tell from his writing and its style that Kuhn didn't want to push anyone's "wrong" buttons, even if they needed pushing, perhaps *especially* if they needed pushing. He wanted his work to find the easiest acceptance it could, especially given that it was sure to be controversial in ways that usually court and find rejection. He was counting on the long term to bring about deep appreciation of the insights he was implicitly pointing at rather more than making explicit, the ones most would find for themselves or more readily come to accept after accepting his explicits. (To make the whole of his intention explicit, however, would have required a rather large library of books.) Newton's rejection of light as waves, we could say, held Fort Kübler-Ross for over a century after his death, acting through easy-to-anger "natural philosophers" who had all too much in common with that first self-proclaimed "lover of wisdom", Pythagoras, in his philosophical reaction to Hippasus's discovery of the existence of irrational numbers.

But the now late Thomas Kuhn never explicitly referred to outright error (or enemy action) as an obstacle or a force to be reckoned with in scientific revolutions. I for one would love to raise/discuss that and many other issues with him. Maybe someday… But for now…

7.2 A Seeming Digression: Poincaré's Chaos and "Approximation"

As was mentioned earlier in **Section 2.5, Poincaré's Further Work on the 3-Body Problem, and Chaos Theory**, Poincaré is considered the founder of modern deterministic chaos theory. Among other things, chaos theory studies systems that, when they are simulationally/computationally started with very slightly—

"infinitesimally"—different initial conditions will have rapidly divergent trajectories. This result is vastly important to philosophy, especially to epistemology and the philosophy of science.

This vision of "(theoretical) chaos" as fundamental to reality and/or its "epistemogeny" might remind us of Gödel's results concerning the completeness and consistency of mathematical theories. He showed that any theory containing arithmetic could not be complete, nor could it be proven to be consistent within the theory. In contrast, "simpler" theories—e.g. certain geometries—can be proven to be complete and consistent.

Poincaré's "chaos" means something similar: no matter how accurately we *approximately* model reality in our science, we will never be able to predict all physical phenomena with reasonable "approximations" to what will in fact happen. The trajectories of some will chaotically diverge from our extrapolations and interpolations based on our approximations to the initial conditions. We must also remember that there will be a "fuzzy chaos spectrum" from "approximate initial conditions will give us correspondingly approximate trajectories and/or computations/-predictions of the trajectories)" to "approximate initial conditions will give us non-correspondingly chaotic trajectories (compared to our computations/predictions of the trajectories)". And that is just taking (Newtonian) gravity into account. If we toss in even a few of the many things we have knowingly abstracted out, e.g. quantum mechanics, relativity and the rest, we only get more chances for "chaos" in the reality we are trying to model, and—possibly distinctly—in our models, which are even quite capable of exhibiting "chaos" that the modeled reality may not in fact display.

- The reader is invited—exhorted—to study, especially in what follows, the important relationship between our older concept of "approximation" and our modernly evolving concept of "fuzzy", especially as regards our budding concept of "chaos".

7.3 Another Seeming Digression: The Map is Not the Territory

- The Map is Not the Territory
  The Model is Not the Modeled

Given how many people are familiar with the now old saw "the map is not the territory", curiously few realize that it (too) has vast application to philosophy, again especially to epistemology/epistemogeny and the philosophy of science. We tend to forget that our scientific theories are the maps and models, not the mapped and modeled territory of reality itself.

- We can think of normal everyday paper maps as displaying rather more static "stick-figure" entities, like the ones we draw/print on piece of paper or a chalkboard, and of models as potentially more dynamic "stick-figure" entities, ones that can be "animated", e.g. graphically with computers, but—more importantly—with our imaginations. We should learn to think of logic, mathematics and science as mapmaking/modeling activities rather than determiners or even descriptions/approximations "truth".

7.4 "Speaking" of Logic

The history of logic tells us that its beginnings in ancient Greece have to do with their debates (using "dialectic", developed to be used in these "dialogues" among two or more people holding differing opinions who wish to afflict the others with "persuasion (not necessarily friendly)", thus winning the debate), made popular then and later by

Plato's *Socratic Dialogs*, not to mention Aristotle and his syllogism approach. (For related comments on "logical proof", see F. C. S. Schiller's "Scientific discovery and Logical Proof", in *Studies in the history and method of science*, C. Singer, Ed., §3, p. 237.) One of the important things to notice is that the concept of "winning" in general, including "winning" a debate, is that it is a step-function, not-winning-to-winning, and we should all know what Fourier said about step-functions and the range of frequencies needed to "approximate" them. It is also important to notice the relationship of step-functions to the ancient sorites paradox (Google, anyone?) and our modernly evolving concept of "fuzzy".

7.5 Approximation, Extrapolation, and Error in Science

If the angular deviation of 2 straight lines from Euclidean parallel is non-zero but very small, few scientists will say that they will not diverge/converge *eventually*, if—and this is essential—*if* the problem is stated in these terms. If we have an electronic device, few scientists will say that it will operate correctly under *all* conditions, again, if the problem is stated in these terms. But we always expect "Science" and its "Laws" and "logic" in general to be "Universal" and "Eternal", which even Emerson would allow to be a "hobgoblin of foolish *in*-consistency".

It must be emphasized that:

- We already know that Newtonian mechanics does not extrapolate well to masses moving with a sizeable fraction of the speed of light (and "far less well" to entities moving faster than the speed of light) which experimental evidence from particle accelerators already shows act non-Newtonianly, and much more like relativity says they will.
- Just because we can't pragmatically measure a deviation within a limited experimental context does *not* mean that outside that context the deviation will *not* become very great, even if it remains *pragmatically unmeasurable because* it is *outside* of the experimental context. This is intimately related to the ever-recurring "scaling problem" which we classically find when we try to make e.g. bigger airplanes, bigger factories, usually any bigger anything, or smaller disk drives, smaller laptops, smaller anything, etc, all of which can be generalized to extrapolation in general, or even interpolation. Science has an unfortunate tendency to leave the local contexts of the validity of its laws vaguely implicit, even indeterminate. It also has the tendency to assume that our normally chosen experimental contexts represent a complete and precise "demographic" of the rest of the real world.
- We know from chaos theory (see **Section 7.2, A Seeming Digression: Poincaré's Chaos**, above) that even the smallest differences in initial conditions—and, dismayingly, also from the errors inherent in the approximations that are themselves inherent in our mathematical models of theoretical systems, and from the errors inherent in our computers' instances of approximations of those approximations and calculations involving them—can rapidly yield indefinitely great divergences in the eventual trajectories and/or behaviors of *many* systems.

  An extremely simple example of this is "critical tuning". This old foggy (an old fogy with heavy cloud cover) remembers the days before television, when we relied on radio. Nearby stations came in loud and clear, usually, but distant stations, the ones we usually wanted, were

> difficult to tune in. The slightest movement of the dial would easily take you past the very narrow band where the station came in somewhat clearly. Early televisions had the same problem, by the way, so they had tuning dials on the back of the set that worked roughly the same way. And that was black and white. Early color television introduced critical tuning on several dials at once.

- Science produces *beautiful* flowers, but all too often they are hot-house flowers.

When the position is put forward that lighter and heavier bodies must interact gravitationally, and that there is an asymmetry that means that the falling rates might in fact not be precisely equal, at least some scientists have responded—and one can guess many others would also respond—that the rate difference is "*so small* that it doesn't really make much difference", some even that it is "so small it doesn't really make *any* difference". This is *dismayingly far* from the most "scientific" attitude one can imagine or hope for. Science is *supposed* to be interested in such small quantities if in fact they are real, or even theoretical, as the falling rate difference is both (or even computational or other artifactual quantities). Scientists neglect that when this falling rate difference is *given enough time and space* in which to act, we can actually begin to measure it pragmatically, or at least notice its effects. Science should always also be interested in small differences, even—or especially—if they are methodological artifacts (a simple classic example of which is round-off error); these, too, need to be studied and accounted for, even if "chaotically".

- Even "small" oversights and "small" differences are important, if one wishes to become and remain truly competent.

We have just seen that standard Newtonian gravity (the theory) is enough to at least derive equations that show that, according to that same theory, the theoretical falling rate difference is—generally, but not always—*non-zero*. In conjunction, the astronomically known existence of Trojan asteroids is visible scientific proof that the falling rate difference is in theoretical and/or actual fact—mostly—*non-zero*. To repeat for emphasis, by Newton's own theory of gravity:

- If Galileo had held a 1 kg mass in one hand and a 2 kg in the other, 1 meter apart, and dropped them *simultaneously* from the top of the Tower of Pisa, at the instant of release the lighter body and the Earth would have accelerated together faster by approximately $5 \cdot 10^{-18}$ m/sec^2 (ignoring, of course, all the usual abstracted outs: air viscosity, wind, buoyancy, electro-magnetic effects, "gravitational anomalies", etc.)

This may not seem like much, but Mercutio caught the gist… "No, 'tis not so deep as a well, nor so wide as a church-door; but 'tis enough, 'twill serve…" Science was and is scientifically wrong—and we need to emphasize that this is wrong post-Newton, 1687—when it says that lighter and heavier bodies fall at the same rate. We can compare this to Kepler's being scientifically wrong—and again we need to emphasize that this is wrong post-Newton—when he put the center of volume/mass of the Sun at the focus of the orbital ellipse instead of the center of mass of the 2 gedanken co-orbiting bodies—an essential difference, scientifically, post-Newton. And when such lighter and heavier bodies have an extended period of time to "fall", as they do in orbit, the difference can show up quite visibly, as it does with Lagrange's Trojan asteroids; the bodies behave as though the approximation of "fall at the same rate" is incorrect, which it in fact is *if extrapolated sufficiently far*.

- ✓ In the 1800s, a similar oversight was made in the science of electricity. Scientists and engineers overlooked for decades that the basic Ohm's Law equations they were using did not allow for current loss by "leakage" through the insulation of their wires/cables. They were using gutta percha, which was a very good insulator over short distances, but as they started trying to use longer cables for telegraphy, over distances of hundreds of miles, the line losses became so great that they had to revise the standard equations. By the time of the first successful trans-Atlantic cables, there was a standard coefficient for line current loss through the insulation to the "ground" or "earth" of sea water. The same situation offered similar surprises concerning inductance when scaling up. (See Paul J. Nahin's excellent bio of *Oliver Heaviside, The Life, Work and Times of an Electrical Genius of the Victorian Age*, 1988, Johns Hopkins University Press, 2002.)

  Scaling—anything, up *or* down, and we even more often seem to forget sideways—will always have surprises ready and waiting for us.

7.6 Abstract Reasoning

Approximations are inextricably linked with abstract reasoning—essentially always occurring together with it—and abstract reasoning is one of the swords that science lives by… and perishes by. Eventually we must realize that *abstraction*—by definition, by *its own* "Nature"—is only capable of letting us reason about the relatively infinitesimal (sub-) map we have drawn (with inherently non-existent accuracy of approximation because we have left out a relatively infinite amount of the pertinent information, even if not seemingly relevant to the other abstractions that constitute what we think we are interested in at the time) of a relatively infinitesimal portion of a relatively infinite territory; i.e. we can abstractly reason only about a merely "infinitesimal" portion of the "absolute infinity" of reality. At least *some* of the *essentials* to the entirety of the reality situation being reasoned about have almost certainly been *abstracted out* (which we may only notice later), thus making our abstract reasoning formally invalid, if not pragmatically invalid as well. (Warning: you should finish reading this section before indulging your initial Kübler-Ross reflexes.)

Any 2 things interact at least indirectly, so (using a mathematical variant of computer programming's "pseudo-code"), for all x and y, $\partial x/\partial y \not\equiv 0$ and $dx/dy \not\equiv 0$, not to mention that for the greatest part $\partial x/\partial y \neq 0$, and for sure $dx/dy \neq 0$ (where $dx/dy = \sum dx/dz \cdot dz/dy$ over all z , i.e. including relative change by any indirect path of any length; yes, I am leaving out important gory details; notice the relationship of all this to our at times oxymoronically oversimplified concept of "causation"). We forget that that business of "6 degrees of separation" doesn't negate all the relatively infinite "n degrees of direct and indirect influences". We forget about Brazilian butterflies when reasoning about hurricanes in the Atlantic. It is a perennial error and failing of science that we do not universally and eternally recognize and realize all this, and try to correct for it.

Ignored here is the warning of—of all people—Aristotle. "Para-summarizing" from the abundant fallacies he enumerated that seem so difficult for people to avoid in their use of logic, Aristotle warned that for logic to be and remain valid, every entity reasoned about by and with that logic must be and remain (throughout the reasoning process) identically equal to itself, at least in all instances of its use in the extended syllogism gestalt. This is sometimes described as a logically essential "Fixity of Terms"; the terms remain "fixed", "unchanging/unchanged" in all usages within an

individual reasoning process. (See F. C. S. Schiller's "Scientific discovery and Logical Proof", in *Studies in the history and method of science*, C. Singer, Ed., §7, pp. 240-2.) This is sometimes modernly para-summarized as: "A ≡ A". (He probably overlooked that this requirement must also extend to what we today term the logical operators, "and", "or", "therefore/implication", etc., but, what the heck.) Since applied mathematics (including logic) and science are both in the position that an abstraction can never be the thing from which the abstraction was… abstracted (we can almost say the same thing about approximation), it is the case that inherently $A \not\equiv A$ and even most probably $A \neq A$ (although still ostensibly allowing the possibility of $A \cong A$, i.e. A is approximately equal to A, more or less, and "chaotically" and "fractally" to boot), and thus, not only applied logic, but applied mathematics and science are ultimately essentially-theoretically invalid, except as maps and/or models whose map trajectories eventually diverge potentially infinitely from the territories we want them to at least approximate roughly. They, too, are also both formally invalid, since some well thought out "A ≡ A"-like principle *must* be recognized as a formal requirement of the foundational logic of both applied mathematics and science, as well as of abstract reasoning in general, thanks to, of all people, Aristotle.

- Science can never truly be essentially or even formally valid on its own strict terms, since the map of abstraction can *never* really be the territory of reality ($A \not\equiv A$), or even approximately isomorphic/analogous to it, can never even map or model more than an infinitesimal portion of the territory of reality, or of its Mother Nature, who, we should be forever thankful, will keep on doing as *She* dinking well pleases. The maps and models we make of the world, or rather of a small part of it, however well they can be said to approximate the territory of that reality, will always have the potential for Poincaré type chaos(es) inherent in them, at best, even if what we are attempting to map or model does not have Poincaré type chaos(es) inherent in *it*. And vice-versa. And the abstraction we per force engage in ensures that we won't even have a map or model that is truly close in terms of an approximation to "*all* initial conditions" of Mother Nature to begin with. We *should* be able to imagine our world filled with Poincaré-chaotic entities that we cannot see because to us they seem merely to be part of the ocean of chaos and chaotic "non-events/non- entities" surrounding us. We cannot map, model or measure them, or even notice them enough as events or entities to think of observing them. Etc.

  Our best reasoning will always be distorted if not flawed by abstraction, so logic is best utilized for showing up inadequacies in reasoning rather than for being a paradigm of reasoning, especially anything like best possible reasoning. Logic can best show us some of *how not* to reason, *not how* to reason.

  We can also note the relationship that abstraction and objectivity tend to increase together, but the utility of either one or both together tends to decrease after a certain threshold has been breached. (See cute comment of the evolution of objectivity and subjectivity on p. 91.) Also, objectivity tends to imply greater universality and eternality, thus implying much higher probability of occurrence, which, in information theory terms, tends to mean that the information content is tending to zero-zip-nada. Irony…

And we need to remind ourselves of another bit of Greek wisdom:

- **"No man ever steps in the same river twice, for it's not the same river and he's not the same man." (One of many variants of a saying attributed to Heraclitus, 535-475BC, or maybe 540-480BC.)**

We can wisely extend that:

- Imagine *n* men simultaneously stepping in different places in Heraclitus' river; we can easily perceive that we can never have the "same river" even once. And by modestly extending that same reasoning we can never have the "same man" even once. This is more than just "seeing a single something or someone from different points of view". And it is more than that our sense of the syllogizeable existence of that "single something or someone" depends on that "point of view". It is easy to perceive that whenever we "thing" (engage in "thinging" by "thinking", at least by our current usual of these), we run into the sorites paradox yet again. But here we run into perhaps a modest variant of "wovon man nicht sprechen kann…" Maybe we will be able to sprechen more on this at some later date, when our thought and language have evolved a bit more.

Just about everything changes with time, so we often wind up with reasoning much like: $a(t=0)$ implies $b(t=0)$ so $a(t=1)$ implies $b(t=0)$… or $b(t=?)$. And what about relativity and *a* in one non-Lorentz frame implies *b* in another non-Lorentz frame? Here Aristotle, Heraclitus, Newton, Poincaré and Einstein all speak to us speak to us of the possible failings of reasoning mathematically or logically (if different) from even close approximations to entities, predicates/properties and/or initial conditions.

- The **more** we (try to) make our logic (try to) universal and eternal, the more we make it universally and eternally un-realistic.
- It is essential to realize that the Poincaré chaos considerations—that even "infinitesimal" differences in initial conditions can yield rapidly diverging system trajectories—applies, not just to arithmetic, geometric, and functional analysis situations, it applies to logic and its application/utilization in reasoning, especially applied mathematical and scientific situations. We need to ask an essential question : if *a* implies *b*, does "approximately *a*" "approximately imply" "approximately *b*"?

But abstraction is the BIG Bad Boy. And our concept of "cause" is one of the most "abstract", and thus too "inference", "implication", "induction", "deduction" and "proof", in both philosophy and physics. As mentioned above, the abstraction is never even a close approximation to the reality it is abstracted from, and the divergence of the abstract system from the real system is all but mathematically and/or logically guaranteed. It's just a matter of how quickly and how much. We are lucky that abstract reasoning seems to work approximately at all!

- **The abstraction that we eulogize ensures that the "dao" or "ding" that we syllogize is not "the true", is not "an sich".**

  And under the circumstances, emphasizing that this is just as true of approximation—of the river and the man or the sea—as of abstraction is entirely in order, fractally/chaotically speaking.

It can also be noted in passing that Aristotle is perhaps the last (well known) logician for whom the "truth" or "validity" of syllogisms depended on their semantic contents. Later logicians have so far tried to make logical "truth" or "validity"

independent of semantics, but dependent only on the syntax, the written "structure", of the… well, "syllogisms" is perhaps no longer the correct term, so "(syntactic) statements". And when "semantic implication", which we can also refer to as "material implication" since the "material" there refers to the real world meanings of the various portions of the syllogisms (including the implication itself), became transformed into "syntactic implication", which logicians proudly proclaimed was much better because it was devoid of any actual "meaning"—except that it could be accorded a logical (syntactic) value of "true" or "false" which *seems* to be vaguely even if misleadingly "semantic". This led to the "paradox of material implication", where a "syntactic implication", devoid of "(material/real world) meaning", could nonetheless seem to paradoxically imply some such "(material/real world) meaning". This sense of paradox especially related to the "syntactic" logical concept that a "true" result could be properly implied by either a "true" or "false" antecedent, and a "false" antecedent could properly imply either a "true" or "false" result. The concept that it was the "material/real world meanings" of statements that made them and implications derived from them "true" or "false" disappeared from logic (and largely from philosophy in general, if we want to be Biercean about it).

- The maps and models we make, however logical, however mathematical, are not and can never be "truth". It is only the territory—the river, the sea—of reality that can be truly said to be "truth", e.g. the "truth" that we must "know" that "will make [us] free". (Remember, to the ancients "(to) know" was a means of sense perception—like "(to) see" and "(to) hear", but more evolved—and the use thereof, not, as it is to us modernly, the "book learnin'"-like product(s) of the use thereof. Any "truth" that we ascribe to our maps and models—or parts thereof such as "statements"—is really only with regard to the rather "subjective" and "relative" relationship these have to the territory of reality. This is why attempts to find objective "truth" through logic (or whatever) have always failed.
- Notice our tendency to abstract out the relevant, the important, and all too often the essential, and the relationship of this to our all too standard "proof by association", including most famously "(proof of) guilt by association", but also "(proof of) innocence by association", even "(proof of) association by association", some variant of which ("association") is about all that's left after the usual abstraction.
- We also unconsciously do something else when we engage in abstraction: abstraction obviously involves an "analytical" part, where as was observed above we construct an abstraction (in part) by leaving things out, but we also (another part) engage in a "synthetical" part where we take the properties abstracted out of context and put them together—i.e. synthesize them—into a "thing".
- ✓ As a digression, the concept that a "true" result can be properly implied by either a "true" or "false" antecedent, and a "false" antecedent can properly imply either a "true" or "false" result reminds me of the concept that a given instance of a "non-black non-raven" acts to inductively prove that "all ravens are black".

7.7 Warning! More Maunderings of the Philosophical Ilk…

Continuing this essential theme of "Approximation, Extrapolation, and Error in Science", we must note that approximation is a necessary convenience to science, but

scientists—with disappointing frequency—confuse their scientifically convenient approximations with "Laws of Science", which (by current and probably unwise definition) do not formally allow for any exceptions, although in practice plethoras of pseudo-formal exceptions are allowed. We also confuse the map of our "Laws of Science" with the territory of Mother's "Laws of Nature". The failure to properly distinguish our maps from the territories that we are trying to map is bound to be a fatal one in the long run.

- **The most fundamental and perhaps the greatest… oversight in Science today is that the *only* possibility for**

"THE ONE TRUE LAW OF SCIENCE"

THAT CAN BE VALIDLY DERIVED FROM

"THE ONE TRUE LAW OF NATURE"

is, and ever will be:

MOTHER NATURE *ALWAYS* DOES AS *SHE* DINKING WELL PLEASES!
(if She *wants* to, that is…)

And we will here remind the reader that:

- **The abstraction—or approximation—that we eulogize ensures that the "dao" or "ding" that we syllogize is not "the true", is not "an sich".**

- Maps of territories are always "false" in *at least* 3 ways:
  1) projections from higher dimensional spaces onto lower dimensional spaces always yield false positives (e.g. 2 distinct points map onto the same map point) and/or false negatives (e.g. 2 non-contiguous points map onto seemingly contiguous map points);
  2) artifactual inconsistencies (the map is made of paper; Mona Lisa didn't really have oil paints all over her face);
  3) abstraction (we leave out most of reality; see **Section 7.6, Abstract Reasoning**).
- And we have a further quintuple-whammy situation here. Even if everything were (more or less) static, we would only be able to slowly (and punctuatedly) converge to a "complete" map of the territory of, to a "complete" understanding of, the relatively infinitely complex system (compared to our evolving capacity for understanding it at any point) that is Mother Nature. But there are at least five more aspects (besides the slow convergence) in this scenario that are not static, but rather dynamic, even chaotic (and often quite fickle about the whole business): the moving target of Mother Nature, the moving target of us, the moving target of the means we use to try to *observe* Mother Nature, the moving target of the means we use to try to *model* Mother Nature, and of course our moving models of Mother Nature, not to mention the moving target hinted at by that hard-won wisdom discovered (yet again?!) in the early days of computer science: "inanimate objects are out to get us". Each of these synergizes interdependently with the others to increase the overall complexity of the compound moving target. We

will later refer to this moving target in terms of "emergent behaviors", a concept that is becoming ever more popular in complexity theory, which we can see has seriously immense application in the situation we just briefly described.

- ✓ It is inevitable that everything we make up in the way of a "Law of Science" will eventually be found to be "un-Constitutional"—"un-Natural"—and was therefore never a "True Law of Science" in the first place.

E.g. we may have a scientific inverse-square force law, but this does not mean that all forces in Nature are inverse-square, or even that gravity is always inverse-square. Whether or not the "Laws of Science" are considered "universal", *approximations*—by definition—can hold only in limited contexts. The limits of these contexts are all too often ignored by scientists, especially considering the ever increasing complexities of our measuring instruments and their peculiarities.

When dropping "higher order terms" from a Taylor series, for example, scientists rarely—or perhaps never—give any indication that they have studied the limits or thresholds beyond which those dropped terms would become large enough to be pragmatically interesting. That they might remain a very small percentage of all the terms is not the essential issue. That the perihelion of the orbit of Mercury *advances*, even if "infinitesimally", that the continents and asteroids *drift*, even if *slooowly*, so that eventually our maps, predictions, and other extrapolations are "wrong", *these* are *essential* issues of science, none the less essential for not being the only essentials, issues that are all too often overlooked. The case of the usually minute differences in falling rates of (the correspondingly minute masses, compared with the Earth or whatever body they are falling together with, of) lighter and heavier bodies and their relation to Lagrangian-Trojan point dynamics is an example of this.

7.8 Swords to Plowshares?!

"Swords" that science lives by were mentioned above, so we will throw in some more quick comments on two important such. Science has become enamored of two "swords", closely related to abstraction: complexity and simplicity:

- ➢ On the one hand we find scientists loving and evolving fantastically and arcanely complex systems, some even putting Lagrange's perturbation theory to shame, to try to approach approximations of the doings of Mother Nature, or perhaps merely to stroke one's own egos. Complex systems can be fun! (with the right kind of complexity.) These systems (the "legitimate" ones), however, easily become so arcanely complex that they are excessively prone to hidden flaws, and fatal ones at that, perhaps especially in the tacit assumptions that go into them, e.g. those for simplifying the complexities enough to predict and compute results that can hopefully be compared with experimental-observational findings. Science becomes, by dint of this very arcane scientific complexity, a Sacred Cow, and correspondingly free from the embarrassing questionings and critiquings that might be put forward by "lesser mortals", the "inferiors" of those who are properly credentialed—scientifically—in the orthodoxy of that particular arcane complexity and its applications. And there is another closely related psychological impetus to continue to evolve increasingly arcane complexity: "TENURE", with its "PUBLISH OR PERISH" and *its* all too often accompanying "OBFUSCATE OR FAIL TO PUBLISH" and many other obvious follow on syllogisms.

- On the other hand, science has a love of simplicity and its at times compelling beauty. We have wildly enthusiastic applications both of abstraction and abstract reasoning, and of "Occam's Razor" type injunctions to abandon complexity and enshrine simplicity, which most often becomes a problematic Procrustean Bed of over-simplicity. Unfortunately, we are often thus compelled to abandon complexity even when that complexity might offer a far better approach to approximation and understanding of reality than the simplicity—most often *over*-simplicity—a Sacred Someone wishes to impose. The credentials of the applier again figure prominently in these situations.
- Our search for a "Unified Theory (Scientific, of course) of Absolutely Everything" somehow manages to serve both these masters.

As scientists, we have yet to come to terms with the fact that the simplest "thing" *in reality*—not in our *abstract view* of reality—is far more complex than all past and present scientific theories put together, including all those that have ever been imagined and/or abandoned. In fact, the word "thing" gives us an important clue, because of the way it is etymologically and essentially related to the word "think": a "thing" is the product of a "thinking", a "thinging" a-"kin" to the "thing". And the word "thin": by the time we have a conceived of "something" as a "thing", it has already undergone that process of abstracting out all but an infinitesimally "thin" portion of "all" that "it" "is" and "does" and "has been" and "might have been" and "might be" and "might become"… all similarly abstracted. A pseudo-quotable: A "thing" is in the "thin" "thinking" of the "bethinker". The target we miss in both our simplicity and our complexity is that they should match "reality", "all" of "reality"… of "Mother Nature as *She* dinking well pleases".

- We always seem to miss the evolutionary possibilities of "symplicity" and "symplexity".

7.9 Occam's Razor, Logic, and Reality

Let us follow up a bit more on Occam's Razor.

Imagine that we have 3 engineers, each with a 2-dimensional drawing of a device the engineers are working on. It so happens that (in general) each of these 3 drawings is inconsistent with the other 2… You have probably made the leap already. They aren't really "inconsistent" if you think of them as being the projections of a higher dimensional entity (here merely 3-D, and, in this special case) in orthogonal directions onto/into 2-D drawings. Or, as with binocular vision, we can say that the reason the 3 inconsistent 2-D drawings are useful is "precisely" *because* they are "inconsistent" and yet all taken together. It would *obviously* be silly in this case to try to decide which 2-D drawing was the "most correct", and to file-13 the other 2, the almost universal throat cutting exercise we are obliged to perform, as Occam's Razor tries to insist that we do, but only in this case because we are in the know ahead of time. In fact, the 3 views all synergize to give us a higher dimensional picture than any 1 or 2 of the 3 can give us by itself/themselves. "Normal science", however, regularly—"normally"—engages in survival-of-the-fittest competitions that eliminate perfectly fine attempts to picture a projection (generally non-orthogonal) of highly multi-dimensional reality onto/into much lower dimensionality maps, models and/or theories.

This same happens in logical situations. Our standard logic says that the statements "the oak tree is to the left of the maple" and "the oak tree is to the right of the maple"

are logically inconsistent. But any child gifted with common sense can quickly see through the Gordian Knot of this logical conundrum.

We want our logic to be "universal" and "eternal", but this example of relative tree position inconsistency denies either one or the other… preferably both. There may be inconsistencies that are undesirable, but many if not most inconsistencies really open a door to higher dimensionality in our models of reality, just as the tree example shows.

Another much simpler example would be that through a given point in 2-space we have 2 perpendicular lines. Then we are told that there is a line that is perpendicular to both those lines passing through that same point. Again, once the matter is put in these terms we can see the fallacy of rejecting the seeming contradiction/inconsistency that, however much it may be such in 2-space, points us toward the higher dimensionality of (at least) a possible 3-space.

- Any mapping/modeling system of logic that can't handle the above-mentioned perpendicular lines, tree, and many other types of "inconsistency" is nowhere near being able to significantly and non-misleadingly help us deal with the territory of reality.
- (Seeming) inconsistency is most often a doorway to higher dimensions. Competing scientific theories are most often different projections of higher dimensional reality onto lower dimensional theories, and no more in a survival-of-the-fittest competition than the 3 orthogonal 2-D drawings that engineers work with frequently. We may eventually find that any set of seeming contradictions can be made consistent by straightforwardly constructing a model of it/them in a sufficiently higher dimensional space.
- The Law of the Excluded Middle and the Law of the Excluded Contradiction are in fact inherently "Un-Constitutional". Logic and mathematics—and abstract reasoning in general—can only give us an approximate model of a highly abstract map of the territory of reality. Any similarity with reality cannot be guaranteed.
- When we simplify in a given instance as per Occam's Razor, we are all too likely to proceed with that simplification as a given, continuing to leave out of consideration those things we simplified/abstracted out in our first instance, rendering them effectively invisible thereafter, no matter how big, relevant or important they start getting. Science, for example, as others do, tends to conceive of cause and effect in overly simplistic terms: a "single" cause for a "single" effect. This is what I like to term the "single active ingredient theory (as opposed to a mere 'hypothesis')". Doctors, for example, only rarely look for multiple "causes" for the often many symptoms. They look for "the" medical "cause". It can be good to ask ourselves: "what is the single active ingredient in an army?" (Anyone even half wise knows that it's the sergeants.)

7.10 Metaphysics and the Supernatural in Religion and Science

Along with rejecting Aristotle, this newly evolving natural-philosophy-science also rejected—*tried* to reject—*all* "metaphysics", reject it as if the metaphysics of religion were responsible for all its ills, even for all the ills of the world. Metaphysics literally translates as something like "beyond the physical (world)". Religion made (and still does) much of metaphysics, *almost* synonymous with that other scientific anathema, the "supernatural", the Kantian noumenal world beyond the phenomenal world, the unseen "real" (but, to scientists, non-existent "unreal") world beyond the mere

physical world, which only the orthodox priesthood was/is considered authorized and empowered to be cognizant of and to act as monopolistically prerequisite intermediaries for. Everyone, especially the common people, had to Accept All This on "Faith", which unfortunately no longer had/has its original Biblical meaning of "being intimately connected to so as to be able to 'know'—in the ancient sense of perceiving—the 'truth' ('and the truth shall make you free')". Rather, the meaning of "Faith" had—as have the meanings of so many Biblical words—been essentially distorted and misinterpreted into a concept of "affirmation (or denial) founded on ignorance, preferably willfully orthodox ignorance". The true "metaphysics" and the true "supernatural" of religion suffered accordingly, as they then came to be "known" by this new concept of "Faith" rather than by the original Biblical concept. This was all obviously antithetical to the newly evolving Greek-philosophy-*natural*-philosophy-science, which *attempted* to reject "metaphysics" and anything "supernatural" *entirely* rather than follow the philosophy of "when wrong, to be put right".

The (acknowledged) fundament of our modern science is "empiricism"—and for some its more forceful statement as "positivism" or "logical positivism"—which forms the basis for our current "scientific method" (however ampolyguous those terms may seem to some). But, if we stop to philosophize about it, we can see that "naturalism", "empiricism", and "positivism" are really just "solipsisms" with their boundaries extended a bit ("haggling about the price…"). In fact, any flat out rejection of "metaphysics", the "supernatural", "other worlds (especially ones not like ours)", or anything of that kind in general will really turn out to be a modest variant of "solipsism". E.g. the term "supernatural" really just points out into the infinite ocean of our ignorance. As scientists—indeed as anyone—we are less than wise to hold that the infinite ocean of our ignorance does not exist, or exists, but only like the "empty set" of set theory, with the tacit yet absurd inference that we are "(all but) omniscient".

The visible spectrum is just a small part of the whole spectrum of light, so most of our world is "directly invisible" to us. But we neglect this in our sense that there is no world beyond "this world", the one that is directly visible to us. We have only seen an "infinitesimal" portion of all the world that we can see even in the visible spectrum. That "infinitesimal" gets smaller if we think of microscopes and telescopes, and the fact that they increase the relative portion of the world that we have not seen. And ultra-sound, if we think of applying it as another kind of sight, makes that "infinitesimal" smaller still. And physicists keep speaking of the minimum number of other dimensions needed to form the infrastructure for our 3-1-dimensional space-time as being maybe 8, up to maybe 11 (as of a few years ago; I haven't kept up). Every day science is busy further expanding the "horizons" of the world that we are starting to know that we have *not* yet seen relative to the world that we *have* seen. That is one of the great wonders of science, that we are starting to get to know more and more about this "ever so much more than we ever imagined… or stranger than we can imagine… or dreamt of in our philosophies".

Yet many "scientists" will somberly, scientifically declare to us that those as yet unseen worlds—including any ostensible "meta-physical" or "super-natural" worlds beyond that already visibly *perceived* by us, *and* their inhabitants—do not and cannot exist scientifically and therefore do not and cannot exist in reality; and, for (controversial) example, there cannot possibly exist intelligences other than us who are interfering in our local existence and affecting if not effecting our evolution.

Hmmm…

- The historically most common methods of proof, "Proof by Ignorance" and "Proof by Lack of Imagination", even when updated to "Proof by Scientific

Ignorance", "Proof by Scientific Lack of Imagination", and "Proof by Lack (or Failure) of Scientific Discovery and Scientific Study" (the most popular examples when I was a kid being "Science proves that ESP and UFOs don't and can't exist"), have absolutely no place in the science I have loved since childhood… "… when wrong, to be put right." These are science at its most dismayingly incompetent.
(Don't get me wrong, "Proof by Religious Ignorance, Proof by Religious Lack of Imagination, etc, are no better. And replacing "Scientific" or "Religious" by anything else—e.g. "Political", "Social" or "New Age"—wouldn't help either.)

- By the way, Kuhn describes the "normal science" stage that science reaches in its development. (I think "normal science" stages in the Kuhnian sense did not really make their appearance until after Newton had started taking effect.) At that stage, that which constitutes the "scientific method(s)" that that particular group of scientists will use, the "paradigms", the "disciplinary matrices", whatever, are all—more or less—agreed upon and decided on a "survival of the fittest" basis followed only then by a "peace in our time" "live and let live" basis. Kuhn does not use the term "Procrustean", but he could have. By the time the "normal science" stage has evolved, "heedfulness" of a *general* sort no longer plays any part. Long gone are the childlike days of trying things to "see what happens" (not merely in the limited Baconian sense. "Eternal vigilance" of a *general* sort has no counterpart in science. Real things that have not yet been fitted successfully to the Procrustean Bed of "normal science" are no longer eligible to have their existence even be acknowledged "normal scientifically". It is only when a "crisis" occurs, during the early part of a "scientific revolution", that scientists *might* be allowed any panic-based freedom to be generally heedful, to look around, grasp at straws, try "new things", and allow these things any kind of entrance into the "delivery room of the realm of the new normal science".
- (Burning need to digress further…) If you have read Kuhn's *SSR*, you may have noticed that he confines his examples to the period in the evolution of science where his concept of "normal science"—which he should perhaps have referred to as "normal natural philosophy"—could be said to have first recognizably established itself evolutionarily. (Importantly, this was *post*-Renaissance, after the European guilds had lost their stranglehold on both technological innovation and to some extent pre-Renaissance scientific innovation, of which there was not much by today's standards.) He just barely refers to what "not yet pre-science natural philosophy" (my terminology) was like and only vaguely refers to the evolutionary mutations that turned it into "pre-science natural philosophy" (again, my terminology). He does not even hint at future evolution other than the alternating cycle of "normal science" and "scientific revolution" that he describes. In particular he does not comment at all on religions having gone through similar phylogenies/-ontogenies, "normal religion(s)", the "structure of religious revolution(s)", and being the major source of pre-historical and some historical technological innovation (predicting the flooding of the Nile and planting/harvest times, discovering medicines, etc), and all that. I greatly miss what insights he might have offered as regards this "comparative religion" opportunity. I also miss what he might have said if he had taken into account the pre-Renaissance guilds that were literally stifling almost all technological and related scientific

innovation. Kuhn, although he is considered a "founding father" of Science and Technology Studies, seems to completely ignore the fantastic evolutionary affect that technology (and its associated "market/mixed economy", "$$$", or related motivations such as warfare) has had on the evolution of science since Archimedes, and, within that sphere, on "normal science" and "scientific revolutions". He completely ignored that science is evolving in the direction—repeating the phylogeny—not only of religions, but of those pre-Renaissance guilds that were strangling innovation and commerce in technology and thus stifling the "not yet pre-scientific natural philosophy evolving into pre-scientific natural philosophy" as well, the death grip of the guilds broken mainly by a combination of the Renaissance itself and the discovery of the "New Worlds" (not just the Americas, but Asia as well, as a source of novelty and as a new marketplace).

STS would do well to add to its repertoire the detailed "comparative religion" studies of the interrelated phylogenies of religion(s) (which were benefiters, benefactors and regulators of technologies early on), the technologies themselves and related guilds (whether of religious or other origins), and natural philosophies going on science(s). We should take warning that the Michael Faradays and the Oliver Heavisides of our time will never be admitted into the "science guilds" that have namelessly taken hold of our now 21st Century scientific communities, no longer like they were in the 19th Century. (Note the difference in the relatively warm reception of Faraday in the early 1800s and the exceedingly grudging reception of Heaviside in the late 1800s, then extrapolate another century.) Our modern Faradays and Heavisides will of necessity need to first become independently wealthy in our still rather more frontier-like technology communities to have even a modest chance to bequeath us their true riches in the realm of science.

So to sum up and make a very long story quite short, science has—or rather most scientists have—*tried* to reject all "metaphysics", all things "supernatural", and who knows what else.

But why the "tried"?

Philosophers in general, though seemingly almost no modern scientists or philosophers of science, know an ironic secret, and that secret concerns the essential difference between "phenomenology" (here the general term, *not* that developed in the philosophy of Edmund Husserl (1859-1938) or Martin Heidegger (1889-1976)) and "theory". Pure phenomenology looks only at the data of the (by definition more-directly-rather-than-less-directly perceivable) phenomena of nature, but it *never* generates any rules, laws, principles, concepts, hypotheses, theories, and/or other dogmas to explain the data. Any such explanatory rules, laws, principles, concepts, hypotheses, theories, and/or other dogmas would be—that's right—"*meta*-physical" (and also—technically—"supernatural"). When any theory (scientific or otherwise) generates explanations (as it must, to be a theory) of any kind for the data obtained (e.g. "scientifically"), it tacitly—and, at least in the case of science and its philosophy, hypocritically—engages in metaphysical speculation. Pre-science natural philosophy started openly disparaging and condemning religious metaphysics starting very roughly with the Renaissance, especially with Newton, and renamed as science still does today universally.

When Leucippus and his pupil Democritus proffered the philosophical concept of vanishingly small atoms swirling in a void, this was pure metaphysics, yet it was more or less accepted philosophically—at the time. But as late as the 1800s, the famous

physicist and philosopher of science Ernst Mach (1838-1916) decried the "atomic theory of matter" as flat out metaphysics and thus unscientific and unacceptable. Even the concepts of "force" and "mass" (as opposed to weight) were considered to be metaphysical. Strangely, though, "energy" was considered by Mach and others to be "directly perceivable" and therefore *not* metaphysical, but rather the fundamental entity in a sort of ultimate scientific principle that they called "energetics" together with its scientific philosophy of "energeticism". And, even though today we have fascinating computer generated images of what we feel quite sure are atoms (obtained using e.g. STM – Scanning Tunneling Microscopy), since we cannot *directly* observe these atoms ourselves, their existence is *still*, *by a scientific technicality*… that's right, metaphysical speculation, maybe even "supernatural". Ironic, isn't it?!

When natural philosophers, such as Galileo, observed cannonballs in flight and described the data obtained from observing the paths they followed as sections of parabolas, they were actually engaging in "soft" metaphysics—we can literally call it "stepping-stone" or "gateway" metaphysics since it leads us on, mathematically and inevitably, to… the "hard" stuff. (If they had put their money on sections of ellipses, they would have been theoretically-metaphysically closer to the mark. By the way, even abstracting data from phenomena—and that includes observation and even perception—must have implicit supporting metaphysics in its/the foundational epistemology.) By the time natural philosophers such as Newton (and others) started peddling the concept of "gravity", they were definitely dealing the "hard" stuff, "hard" metaphysics. The invisible "action at a distance" inherent in the concept of gravity made it just as metaphysical, just as supernatural, as that which they were resoundingly *trying* to reject in religious dogma. (By the way, even Newton himself abhorred the "action at a distance" concept necessary for his theory of gravity, for just these reasons. A great many breathed a sigh of relief in the 1800s when the metaphysical concept of "action at a distance"—which sounds religious ("the Divine Hand…")—was finally replaced by the metaphysical concept of a "field (at a distance)", which "field" itself then somehow causes the "action at a distance" with its associated "lines of force", thus sounding suitably "scientific". We should all be grateful that Relativity discarded all this "action at a distance" hooey and replaced it with a much saner "curving space (-time?!) at a distance".)

- ✓ Digressively: it is no mere coincidence that the characterization of the flight paths of cannonballs as (sections of) parabolas corresponds to the pre-Newtonian idea of an Earth that does not move, e.g. under the influence of the force of gravity supplied by a falling apple, not to mention corresponding to the gedanken concept of a physically impossible "uniform gravitational field", whereas (sections of) ellipses correspond to the Earth also being accelerated by the force of gravity (and the bit about inverse-square, etc), as per Newton's theory.

When "natural" philosophers dabbled in particle theories of light versus wave theories of light, and later, as scientists, married the two theories, they were tacitly (and quite probably unconsciously) evolving the inescapably inherent *metaphysics* of science, while still *verbally rejecting* metaphysics in general, where the only metaphysics they had ever known by that name was the metaphysics of Religion, which had come to be an instrument used to deaden the souls which religion was—*supposed* to be—responsible for re-enlivening and even resurrecting. But these new Renaissance intellects could not publicly admit that they were actually just *replacing* the metaphysics of religion with their *own* (punctuatedly evolving) implicit

metaphysics, and replacing what was previously *theologically* "*super*-natural" with their newly *theoretically* "natural" (likewise punctuatedly evolving), probably because they felt they could not expose any weakness in their (still mostly tacit) rejection of religion, and by association, their explicit rejection of Aristotle and metaphysics in general. The essential difference between Newton's gravity and Einstein's gravity, we can remind ourselves, is far more a difference in the metaphysics of their respective theories than in their phenomenologies.

- Ultimately, although our modern science seems to have rejected metaphysics entirely, it has actually (mostly only) rejected the metaphysics that science has associated with religion, generalizeable to a variant of that all too Universal and Eternal "Not Invented Here". Science has been quite happy to go along evolving an ever more sophisticated "science-based metaphysics", although it carefully does not allow the use of the word metaphysics to name or describe it, even though that is what it is.
- So, given the explicitly complete rejection of "metaphysics" by "scientists", it's quite ironic that (for example):

  Our concept of "Universal Laws of Science" is a *completely* metaphysical concept.
- Given that metaphysics is such an essential part of any theory (even if invisibly and unappreciatedly), rather than denounce metaphysical speculation, all scientists should feel called upon to "get good at it". We should come to think of metaphysical speculation as extended exploration through gedanken experiments.
- When we think of science as opposed to religion, we should not forget that business about just who is authorized to be cognizant of and to act as orthodox intermediaries for that unseen "real" world beyond the mere physical-material world, where now—instead of "God"—the "Ultimate Laws of Science" and the "Ultimate Theory of 'Life, the Universe and Everything' have their "Valhalla of Science".
- We can note, with especial irony since science is evolving so as to Darwinianly "displace" religion—or perhaps more Oedipally to "take its place", since "Theo" is from "Theus", the Biblical Greek word for God:

  "*THEO*RY" and "*THEO*LOGY" are really *spelled the same*.

One of the oldest and greatest of wisdoms is that when we are speaking about reality—as opposed to some abstract mathematical systems—one can never truthfully say or competently prove that something is "impossible" in reality, whether that something is "imaginable" or "unimaginable". Reality is just too infinite, too ever-changing, too varying… even too fickle. Even that infinitesimal bit of reality that we are *not* blind to and *not* deaf to has countless wonders beyond any "dreamt of in our philosophies". We already know from the advances we have made in science in just the last few hundred years that there is a sometimes overly subtle difference between the "supernatural" and the "as yet unknown natural", that a good portion of the supernatural of yesterday has often become the scientific or even common sense natural of today, as natural as… as walking on the moon. This progression, though not strictly guaranteeable in advance, is quite obvious if we pay attention, if we repent of our heedlessness. The "supernatural" is really just "the natural" that lies in wait for us, for us to grow up and open our new eyes, new ears, and new minds.

- When science rejects "metaphysics" it engages in unrecognized hypocrisy. When science rejects "the supernatural" as scientifically impossible it engages in—what should be recognizable as—scientific incompetence.
- In order to scientifically reject "the supernatural" or to otherwise say that something cannot exist in the absolute, and almost completely unknown to us, infinity of reality, science must tacitly invoke the "Axiom of Omniscience" (and its litter-mate, the "Axiom of Infallibility"): "what we *don't* know *just ain't so*." It relates to the scientific oxymoron status of an "unscientific fact".

If you think this is an exaggeration, just remember how ever more often our ordinary (non-scientific) knowledge, even our ordinary (non-scientific) common sense, is being rejected out of hand as "unscientific". Think how often you hear—or say: "you can't say that; that's not scientific!" As noted above, an "unscientific fact" is almost regarded as an oxymoron in our society, and it *is* regarded as a scientific oxymoron in the scientific community. So let's add a tacit "Axiom of Infallibility" (regarding Anything that Science Claims to be Scientific or Unscientific in matters of Scientific Faith and Morals), similar to the kind the Church decided it needed as explicit dogma (credenda, in addition to the traditional tenenda) in the late 1800s, in what had become all too obvious were the waning days of the Inquisition. We can certainly see Science Oedipally trying to take the place of Religion, even if the "disposition" of Laius consists mainly of dogmatically declaring him "Un-Scientific" and banning him rather than burning him at the stake, or whatever.

A well-known counterexample to the term "unscientific fact" being oxymoronic is the by now seemingly ancient (to scientists) bit about whether bumblebees can fly. In the late 1940s and early 1950s, when I was just a lad, scientists were a bit more humble. They would publicly admit that bumblebees could in "*un*scientific fact" *fly* even though in "scientific fact" they could *not*; the best aerodynamic models of that time, according to aerodynamicists of that time, predicted solidly that bumblebees could *not* fly. This admission was almost certainly forthcoming partly because bumblebees were just too shameless in their pursuit of happily but "unscientifically flying" in public, and science would have tarnished its less than deserved reputation too much by trying to deny that particular "unscientific fact", as it does with so many others. Well, science has made advances in promoting bumblebees from "unscientific flying" to "scientific flying", but as of this writing (late 2009), bumblebees still have to content themselves with the former. These days scientists prefer to "no comment" flying bumblebees, especially as relating to the *science* of aerodynamics. That's a slight exaggeration, as if you couldn't guess; I recently watched a very interesting show on the Discovery Channel about studies of bumblebee wing motions using ultra-slow-motion photography. Fascinating!

It helps if we combine some of our budding complexity theory—some are enamored of the term "complexity science"—with some ancient yoga of wisdom. Our complexity theory has started introducing us to the essential concept of "emergent behaviors". (It doesn't really matter, for our purposes here, what these behaviors might emerge *from*, or *how*.) We need to combine this with an ancient yogic wisdom that one must always be mindful of at least three things: (not just) the "observed", (but also) the "observer" and the "means of observing". We combine these by noting that the "emergent behavior(s)" concept does not just apply to the "observed" or "observable" system, *but also to the "observer" and the "means of observing"*. Any behavior that emerges—makes its *debut*—in a system without corresponding emergences of-and/or-in possible observers and of-and/or-in possible means of observation is likely to feel unappreciated, and no debutante likes *that*. As babies, we keep our eyes closed for

weeks, just waiting to open them and turn that to *us* supernatural—and, to *some* extent, intelligently designed and run—nursery (where the "nurses" and "mothers" are "supernatural" entities) into a natural—and, hopefully, even if differently, also intelligently designed and run—home.

It should be noted emphatically that emergent behaviors are not necessarily linear, ordinal, or hierarchical. It should also be noted emphatically that emergent behaviors need prior (though not a priori) and also ongoing "mergent behaviors". In general there is no single "line of successive emergences", as we sometimes think of humans being the ultimate emergence of evolution or someone's intelligent design. Emergent behaviors punctuatedly evolve in "all directions", "every which way including loose", just as Nature does in general. They experience the urge-to-merge-evolve-emerge-evolve-remerge… "all over the place", topologically speaking.

Perhaps nowhere does this show up more clearly than in the arenas of science and religion. The question "what is the scientific method?" is actually one of the most ampolyguous in history, the term "scientific method" often seeming to have almost more definitions than there are scientists, second only to the question "Is there a God?", which is distinctly *The* most ampolyguous in history. "The scientific method" is distinctly a whole class of emergent behaviors that have their existences anatomically and physiologically—not to mention phenomenologically and noumenologically—spread out across the observers, the means of observing, and the observed(s), emergent behaviors of "the scientific method" corresponding—somewhat loosely—to uhh… corresponding interacting emergent behaviors in those same three… and who knows what else.

In my honors chemistry class in college—this was back in the early '60s—we discussed how we could not yet derive chemistry from physics, not even from our best quantum mechanics let alone from nuclear physics; i.e. we could not scientifically prove, *starting from physics*, that chemistry even existed, or biology. It was also mentioned that just about everything that seemed to be a science, and seemed to be based on other sciences—like chemistry was based on physics, biology was based on biochemistry, and psychology was based on biology—could *not in the foreseeable future be scientifically derived* from those other sciences. But we all managed to feel pretty sure that some day we would be able to do so… uhh… scientifically sure.

Science itself is multiple composites of emergent behavior(s), emergent behaviors in us as observers and in our means of observation, behaviors that have not yet emerged to a level of sophistication—"Sophia", remember, once meant "Wisdom (personified)"—that allows it and us to competently scientifically examine let alone pass scientific judgment on such things as "Intelligent Design Theory" or "Creation", themselves "emergent behaviors" of observer, means of observation, and observed. Science's "Religiously intolerant" attempts to do so, especially its attempts to disallow even *discussion* of "Creationism" along with "Evolution" in our *schools* are distinctly "Good Science, in the worst sense of *The Word*". "Proof by ignorance", "proof by lack of imagination", "proof by refusing to look through Galileo's telescope", "proof by Inquisition", and "proof by denial and dismissal" in general, together with science's attempts to deny the name "theory" to anything that does not lie in the domain of science's "dogma"… well, I must here reiterate that "THEORY" and "THEOLOGY" are "spelled the same". Bumblebees do not fail to fly just because it has been "scientifically proven" many times that they cannot fly, or because they cannot be "scientifically proven" *to* fly. Life does not fail to exist just because we cannot *even begin* to *scientifically* prove that it exists.

"Eppur si muove…"

A crude and ill-defined example of emergent behavior in the "observer(s)" might be "scientific competence", which *enables* those observers to *allow* themselves to look through Galileo's telescope, or—more to the point—the telescope of someone not yet "scientifically kosher", and to do so with reasonable success of various relevant kinds. An example of emergent behavior in the "means of observation" might be a scientific instrument, like Galileo's telescope, a new experiment and/or Kuhnian style "paradigm" of experimentation, perhaps "quantum chemical", or astronomical observation, a new "paradigm" of relating to the real world, whether previously unseen or "seen through other eyes". And an example of emergent behavior in the observed might be seeing—through Galileo's telescope—4 white spots seeming to move in close proximity to a big whitish spot that one takes to be the planet Jupiter. That last has at least two sides: the behaviors (white spots or moons) may have just emerged (formed by accretion), or they may have emerged some time before and our now detecting them is itself the only emergent behavior.

- Although they obviously do not emerge in "lock step", the emergent behaviors in and of observer, means of observation, and observed, are all interdependent on each other, and also tend to synergize and "serendipitize" vitally. Emergent behaviors similar to these scientific examples historically also happened—and still happen—in religion, which is itself still "punctuatedly evolving".
- We can especially note that if sufficiently complementary emergences have not yet occurred in all three categories of Observer(s), Means of Observation, and Observed(s), then somebody's (logical?) positivism will be sorely lacking in foundational episteme. And this doesn't even treat the serious problems that occur when fundamentally differently emerged observers try to use the "same" emerged means of observation on the "(same emerged) observed", or… well, think of all the permutations and combinations involved in just the overly simplified real world systems of emergent behaviors relevant to science and religion hinted at here. The most prominent examples in our 1st decade of the 21st Century scene are the "Science Wars" (and these "Wars" are multiple and often highly interrelated phenomena), which includes the "Anti-Evolution vs Anti-Creation/Anti-Intelligent Design" donnybrook.

Here is a related bit (quoted without permission, as of this writing) from T. Kuhn's *SSR* that is very interesting in this context:

- "To a greater or lesser extent (corresponding to the continuum from the shocking to the anticipated result), the characteristics common to the three examples above [concerning the evolution of Leyden jars] are characteristic of all the discoveries from which new sorts of phenomena emerge. Those characteristics include: the previous awareness of anomaly, the gradual and simultaneous emergence of both observational and conceptual recognition, and the consequent change of paradigm categories and procedures often accompanied by resistance. There is even evidence that these same characteristics are built into the nature of the perceptual process itself."

  Thomas Kuhn, *The Structure of Scientific Revolutions*, any edition, p. 62.

Science is not nearly our only emerging-evolving system of observing and knowing, especially not our only possible such, however much it may deservedly be currently in fashion, however much success it may arrogantly be having in an importantly emerging-evolving field where it is the only competitor that it allows to

officially compete. What we term "Intelligent Design" is the result of emergent evolution that diverged (over millennia, and nothing like a "simple" divergence) along significantly different lines than our "Modern Science". (Newton, as did many other natural philosophers, believed he was finding the "Laws" behind God's Intelligent Design of Nature, e.g. in his theory of gravity.) "Science" and "Scientists", both as "observers" and "means of observation", have just not emerged-evolved sufficiently and/or "in the right way" so as to allow "Scientists" to "scientifically observe" the same things that "Intelligent Designists" consider their important "observed(s)", and vice-versa.

Although it might seem like too much of a heretical digression to "some", it is important to note that our "scientific" rejection of "metaphysics" and "the supernatural", e.g. of "Intelligent Design" and "Creation", is completely unscientific, and not worthy of our efforts in those directions. "Anti-Creation" and "Anti-Intelligent Design" are *just* as anti-scientific as "Anti-Evolution", literally a mockery of science. And, despite the fact that even the much more scientifically sophisticated *neo*-Darwinist concept of evolution is still quite scientifically naïve, or perhaps you would prefer "not fully formed", "Anti-Evolution" is distinctly "Anti-Religious", "Anti-Intelligent", literally a mockery of God and creation.

- Although this is an over-simplification, it is the "Anti-" in "Anti-Evolution", "Anti-Creation", "Anti-Intelligent Design", and "Anti-Just About Anything" that gets us into such terrible trouble, not just "philosophically", but in reality. "Mother Nature *always* does as *She* dinking well pleases!" We need to keep putting ourselves right by reminding ourselves of that essential ancient wisdom that one can never truthfully say or competently prove that something is "impossible" in reality, whether that something seems to be "imaginable" or "unimaginable".

The "emergent behavior phylogenies" of those who study "evolution" and those who study "creation/intelligent design" have been significantly different, and this needs to be taken into account if we wish to have sanely evolving intelligent dialogs on these and future matters. We all need an infinite ("divinely/intelligently selected") succession of emerging-evolving doses of "you shall know the truth and the truth shall make you free." There are lots of "observers", lots of "means of observing", and lots of "things to observe" just waiting to emergently behave through our skulls, however much thickness has so far emerged in them. "The sun is but a morning star…"

7.11 The "Procrustean Bed" and "Scientific Fact"

Procrustes was a bandit (some say also "king") who came from Attica and set up shop in the hills near Eleusis, a coastal town between Athens and the Isthmus of Corinth. Procrustes had an iron "bed" which he "invited" wayfarers passing through his kingdom to lie on—one can presume to be "measured on", "found wanting", and then "fitted to". Any wayfarer parts that extended past the ends of the bed were cut off. If insufficiently long, the wayfarer was *stretched* to fit. Sound familiar?!

The name "Procrustes" means "stretcher", and the rack that we still associate with the Inquisition (still with us today, even if known by different names, and associated with seemingly different people and organizations) was probably "inspired" (though not divinely in the best sense of the word) by the "bed" of this historical-mythical Procrustes. Some accounts use the name Damastes ("subduer") or Polypemon ("many harmings"), adding yet more dimensions to the myth.

If you saw "*The Sting*" with Paul Newman and Robert Redford, you will remember the bit about "running a wire". Procrustes is said to have spied wayfarers approaching from afar, and "adjusted" his bed accordingly. According to some "authorities", he used a hammer to beat thin the *too* short until they became long enough (how this fits with "stretcher" is anyone's guess), or a handsaw to cut off the parts of the *too* long until they became short enough. Some accounts say he had *two* beds, one short, one long.

No wayfarer had a chance—sound familiar?!—until Theseus came along. Theseus (who was apparently tallish, sometimes described as "stout") fit Procrustes to his own bed (the short variant) and cut off *his* head and feet, mythically ending the Procrustean reign of terror. It is no mere accident of linguistics that the name "Theseus" seems so similar to "Theus", the Biblical Greek word for "God".

This myth, more than most, just plain old invites "application". The term "Procrustean Bed" has been a staple of analysis and commentary—social, political, psychological, and satirical—for millennia. Both the myth and the term are as insightful—and unfortunately as applicable—today as ever. Make a list: "Discipline", "Self-Discipline" (!), "Political Correctness"… the list seems endless… In particular, science makes an all too easy target for said application. I don't know of anyone having used the term in the Science Wars, which I find quite surprising. But one can definitely see the concept operating when we look closely at our usual science in the mirror of "The One true Law of Science". (**See Section 7.7, p. 102**.)

The message of this is that we are all too rapidly approaching a time when "facts" that do not "fit" a (still accepted) scientific theory will not merely be considered "unscientific", but will also be considered "not facts". All of us, *especially* scientists need to guard against this future.

7.12 "Science Wars" and "Ablative Shielding" (and Science's Implicit "All But Infallibility")

In the current "Science Wars", the critics of science tend to recount the philosophical, social-sociological, ethical, spiritual, moral and even political failings of science, all of which are rejected by science as "unscientific" bases for criticism—of science, at any rate. Scientists, in turn, tend to recount the "scientific" failings of science's critics (academics in the "soft" social sciences and cultural studies, feminists, New-Agers, STSers, etc).

But still all-but-universally overlooked is the fundamental fact that science has a *long history* of failing *on its own terms and on its own turf*. Aristotle was a modern scientist of his day, as was Ptolemy, and as of course were many others. As (some of) these theorists and/or their theories fell into "scientific disfavor", the worker-bees of punctuatedly evolving natural-philosophy-science kicked these now unneeded drones out of the hive into the cold of the winter of their discontent—or some such. Like footsteps of the past on that journey of a thousand miles, they have served their purpose, that of advancing science from whatever it was before to whatever it is now… that is *also* about to be left behind. They are now history, even an embarrassment, cut from the team by a subtle variant of Occam's Razor, downsized. Thus (the implicit continuing all-but-infallibility of) science is maintained.

The space program has serendipitously endowed us with a special term for this type of process. The early space capsules had heat shields of a special design that would protect the capsule from the extremes of re-entry temperatures and heat. These heat shields were made of a special material composed of a large number of special (potential) "flakes". Upon re-entry, as the surface of the heat shield is heated by

friction with the atmosphere, these flakes, heated to extreme temperatures, take the heat away from the heat shield before it gets too hot, and thus take excessive heat away from the capsule and the astronauts as they… "take the fall".

- DEFINITION: "Ablative Shielding" – a "heat shield" that functions by letting "flakes" "take the heat"… and "take the fall".

If this sounds political or even universal—a la Dilbert—as well as scientific, it should.

When science changes so much that the old paradigms and their champions are too much of an embarrassment—so much so that people might start questioning (the all-but-infallibility of) science as they eventually did the rather more explicit infallibility of religion—science, like so many others, practices a variant of "ablative shielding" by casting out "flakes", "fall guys", who, "falling from grace", also "take the heat"… away from science and its remaining scientists, and their paradigms. Thus, Aristotle, Ptolemy, and many others were ejected from the pantheon of the "quantinuously", "punctuatedly" evolving "modern science", even though they had once been leading "modern scientists" in their day, and often for many days after. Kepler, on the other hand, is still a close enough approximation to keep in the scientific pantheon—for now:

- Kepler is lucky that the center of mass of the Sun is still (implicitly) considered to be *very* close to the barycenter of the Solar System, or—scientifically, post-Newton—he'd be *outa* here.

Science deals with its *critics* in very much the same way, and all too often with "righteous"—"self-righteous"—anger. Scientists have come to be afraid of any kind of shift—paradigm or otherwise—that might disenfranchise them from their developing-evolving and proliferating preeminent position of rule-authority-power, about which the Bible unambiguously and *strictly* warns us *against* (1st Corinthians 15:24). The King James uses the quaint old English euphemism "put down", but we should not be deceived about what God—or "some such"—meant and *still* means by that. Science, like Religion before it, seems to want much more than just the freedom to… uhh… "pursue science freely". Science seems to want rule-authority-power that extends well beyond science into governance and politics, into the control of our everyday lives that religion used to have, and still has to a great but almost invisible extent. "Two houses both alike indignity, in fair Verona where we lay our scene…" (Yes, Bill knew he was playing with fire.)

If the Founding Fathers were alive today, they might very well also be shouting warnings concerning "the separation of Science and State". Those who truly love science often feel an all too real terror about of the state and the fate of science's soul. We might even be reminded of Thomas Jefferson's "Indeed, I tremble for my country when I reflect that God is just, that His justice cannot sleep forever."

7.13 "Science Wars" and "Punctuated Scientific Revolutions / Evolutions"

There is an impending shift in the relationship between science and the world, somewhat like the "paradigm shifts" that Thomas Kuhn wrote about in his "*The Structure of Scientific Revolutions*" (University of Chicago Press, originally 1962, with its 2nd edition in 1970, as of this writing in its 3rd Edition of 1996). As mentioned earlier, the most important paradigm shift—a true apocalypse for some—will be the "meta-paradigm shift" of a conjoining of the "Science Wars" with "punctuated scientific evolutions". The real revolution that this different falling rate result will encourage has to do with the terrors mentioned above. It is not a source of terror that

Newton's law of gravity will be displaced and replaced, which in any case it won't be, at least not due to the falling rate result presented here.

- **One source of the terror that *will* give impetus to a new and different kind of scientific revolution—"punctuated scientific evolution"—is the one repeated so often here, that, not only did *Newton* fail to note the rather obvious falling rate difference that his own theory predicts, but *every* scientist since has failed to do so, as well, including *many brilliant physicists*.**
- **Another source of the terror is the "righteous/self-righteous" anger that scientists all too often express toward those who try to point out such… oversights. There is a significant pattern of scientific… oversights, and there is a significant corresponding pattern of scientific anger and renamed-repackaged-remarketed-remerchandized Inquisition.**
- **Yet another is that science all too often seems to be seeking to take the place of religion, a quasi Laius-Oedipus situation, and to have religious rule-authority-power replaced completely by scientific rule-authority-power, despite the absolute warning against such found in 1st Corinthians 15:24. This shows up when, for example, "revealing the truth" is considered an act of "heresy".**
- **This is science and its essential scientific integrity failing at their most fundamental levels, theoretical, philosophical, intellectual and spiritual.**
- **This is science failing in its own territory and on *all* of its own essential terms.**

The current heating up in the last few decades of the millennia old "Science Wars" is still based mainly on large numbers of people—including many scientists—finding that science fails on philosophical-ethical-social-spiritual-moral grounds, e.g. regarding cultural bias, or the epistemology of "how we know what we think we know", which of course science does even though it rejects such grounds as… "unscientific". Add to this the terror we should all—*especially* scientists—feel at the thought of people—especially *ostensibly competent* scientists—becoming angry when science is critically questioned, even when critically questioned about such an accepted fundamental result as the equal falling rate finding of Galileo.

The real revolutions—punctuated evolutions—are going to be in world-community awareness of this serious ongoing failure of science, and relatedly in our science education, and we could say in our meta-science, how we *all*—not just scientists—*do* science and *do* science education (and more subtly, *don't* do them). The questions are inescapable:

- If science has overlooked something so theoretically obvious for over 3 centuries, and so astronomically obvious for over a century, what *else* is science… overlooking?!

Aristotle's role in this can remind us of *Greek* tragedy: what fatal flaws is science burdened with *and*—a la Oedipus, killing the father (religion) and taking his place—burdening our world community with?! Talk about "Science Wars"! What will happen when people become aware that science is so decidedly and so egregiously failing on its own terms and on its own turf?!

There is sure to be a meta-paradigm shift e.g. in our communally accepting the actual, even the inherent, fallibility and fragility of science. A subtly related idea is

expressed in (this gets a bit complicated) Chet Raymo's *book review* in *Scientific American* (September, 2000) of Steve Fuller's then hot off the press book *Thomas Kuhn: A Philosophical History for Our Times*, University of Chicago Press, 2000. (Steve Fuller is an Anglo-American sociologist-philosopher who in recent decades has become well-known in the field of Science and Technology Studies, still itself—like science—in the conception stage, often a bit naïve, and all too often driven by the SPECTRE of publish-or-perish, but overall much needed for our common future, just as science itself is, and even religion—which also has its share of naiveté and its own variants of publish-or-perish. See *The Handbook of Science and Technology Studies*, 3rd Ed., 2008.) Raymo wrote:

- "The paradigms of normal science are not the ideal form of science, he [Fuller] says, but rather 'an arrested social movement in which the natural spread of knowledge is captured by a community that gains relative advantage by forcing other communities to rely on its expertise to get what they want.'"

  [Chet Raymo, quoting Steve Fuller]

That has a "Medieval Guild" sound to it.

- ✓ By the way, I have a different take on Thomas Kuhn than does Steve Fuller, prolific author of (among other books) *Thomas Kuhn: A Philosophical History for Our Times*, University of Chicago Press, 2000; *Kuhn vs. Popper, the Struggle for the Soul of Science*, Columbia University Press, 2004; *Science vs Religion? Intelligent Design and the Problem of Evolution*, Polity, 2007. Fuller seems to class Kuhn as authoritarian and Kuhn's "normal science" and "paradigm shift" business as *pre*-scriptive, which I don't understand at all, having read the different editions of Kuhn carefully, at least twice. I found Kuhn rather distinctly *non*-authoritarian and his "normal science" and "paradigm shift" business as over-simplifiedly but insightfully *de*-scriptive. (Kuhn was *de*-scribing them as *considered pre*-scriptive *within* the particular "normal science", an essential difference.)

  In fact, I find Kuhn, despite his limitations, more insightful concerning "scientific revolution" than Darwin was concerning "evolution". Both their insights need to be extended quite a bit in the future, but easily can be so extended and deserve to be remembered. Kuhn's critics sometimes "accuse" him of "relativism" and "irrationality", which descriptions, when put in the form of an "accusation", suggest dysfunctionality. These accusations often give the distinct impression that these critics have not even actually read Kuhn, another reminder of how similar Science and Religion have become.

  Digression 1: Critics also complain that Kuhn's fans/readers *too* freely use, adapt and/or bandy about Kuhn's theses with regard to other "fields". I would like to take a shot at this. Kuhn's *SSR* could easily be applied quite functionally and insightfully to the current stage that Modern Religion has reached in its evolution, which it mostly reached perhaps millennia ago. I point this out as yet another way that Modern Science is evolving to become more and more like Religion, perhaps unconsciously, but "by Design".

  Digression 2: Both "relativism" and "irrationality" have been widely criticized falsely, which is not to say that there are no valid criticisms of either. Truth is a "thing" of reality, and an "absolute thing" only of "absolute reality (whatever that is)". Truth is a thing of maps *only* by some

kind of "association", perhaps "utility" because of "approximation", which would of necessity be "relative". If "irrational thinking" is to "rational thinking" as "irrational numbers" are to "rational numbers", what then is "irrationality" with regard to "reality"?! The famous 19th Century mathematician Leopold Kronecker (1823-1891) thought "irrational numbers" did not even exist, but most mathematicians consider his view that of an otherwise fine mathematician who had a depth he liked to visit a bit too often.
End Digressions 1 and 2.

But after reading his *SSR* yet again with great care, I find that if anything Kuhn is overly engaged in "absolutism" and "rationalism", and even in "totalitarianism" and "establishmentarianism", first, in the way that he does not explicitly or implicitly recognize anything (important) in science and its punctuated evolution that is taking place *outside of* his restricted but well-presented set of concepts, and second, in the way that this restricted view *suggests* "prescription" rather than mere "description" *because* of the way it is so tacitly restricted. (Being well-presented makes it easier to "… oversight" deficiencies such as these.)

My criticism here of Kuhn's "thetical" "totalitarianism" and "thetical" "establishmentarianism", however, actually applies quite a bit more to almost all other researchers-writers in the fields of science and of STS (including the history and historiography of science, and the philosophy and "philosophiography" of science), and even his tacit restrictions and limitations in his descriptions of "normal science", its "paradigms", and its "revolutions" fail to truly convince me of Kuhn's (dysfunctional) "relativism", "irrationality", and all around "antidisestablishmentarianism" (to help separate the goats from the sheep).

Some things are so complex and in such a way that they can *only* be *over*-simplified, especially when treated in a finite forum.

Now, I love over-simplifying as much as anyone. It sure beats writing or verbalizing *The Decline and Fall of the Roman Empire* ("scribble, scribble…") every time you want to get a "symple" point across. And I find Fuller's writing to be insight inspiring, filled with "a-ha"s and "oh, yeah, that's something to think about" even when I totally disagree with him, which is far more often than not, depending on the topic. For example, when writing about Kuhn—especially when comparing him with Popper in *Kuhn vs. Popper*—Fuller seems as if he does more than just over-simplify the boat, he seems determined to miss it altogether. As a renegade relativist in an absolutist world, I only find myself agreeing with missing the boat altogether in just a very few special cases, like, say, the Titanic. Surprisingly, Fuller did rather better in his *Science vs Religion?*

On the other hand, I first found and enthusiastically read Kuhn's *SSR* more-or-less around the time the second edition came out, of which Fuller writes: "Most commentators rely on the second edition (1970), where Kuhn begins a bewildering tendency to retreat from any radical-sounding claims." (*Kuhn vs. Popper*, p. 129.)

So… maybe my ducks carelessly imprinted on this second "sound the retreat" edition. (I just found a used copy of the 1st edition on Amazon and

ordered it. I will read it listening for those "radical-sounding claims". Update: I received it and finished it, listening for "radical-sounding claims", then re-read the 2nd edition, listening for "sound the retreat" noises. Kuhn definitely tried to back off from his term "paradigm" and use other more well-defined terms, but it's also obvious that in the decades since the 2nd edition his fans have decided to stick with "paradigm" even if his use of it made the term ambiguous in the way his critics often claimed. The ambiguity was/is not dysfunctional in the contexts to which *SSR* and "paradigm" are importantly relevant, and is perhaps itself serendipitously insightful in a way that Kuhn failed to positively appreciate.)

One of the reasons Kuhn is so popular is that so many look forward to "Scientific Revolutions" of the kind he analyzes. They see Science—besides as every day becoming more and more like Religion—as every day becoming more and more like the Medieval Guilds that had evolved over the pre-Renaissance centuries to the point where they were literally strangling Europe and stifling almost all creativity in the "arts and natural philosophies" as well as almost all economic and technological activity and the evolution thereof. But, enough of this for now…

It will be more difficult for science, as religion has also found throughout history, to maintain and advance its status, to promote its implicit "practically all-but-omniscient, practically all-but-infallible" image and its explicitly "the only *real* way to go" image, *if* it admits to… oversights, or even to the usual, heretofore seemingly standard "Science Wars"-type ("non-scientific") failings. And science will be forced to admit to at least one truly scientific failing: Newton's Great… Oversight, which still extends itself into our "modern science" and modern science education. "Science Warriors", those who seek the betterment of science in the best sense of the word, will be able to hold out this example of science failing egregiously on its own terms:

- **So simple, so obvious, but our best science and our best scientists *still* can't even get *that* right!**
- **If science has been failing at something this simple and this obvious for over 300 years, what *else* are they missing?! what *else* is science failing at?**
- **And—especially—how does this affect our society and it's ever increasing dependence on science and science education?!**
- **And for *gosh sakes*, why do "scientists" get so *angry* when this kind of thing is pointed out?! Maybe we'd better start hoping really hard that the Intelligent Designists are right!**

Such a world community recognition of the falling-rate-difference… oversight could very well help stimulate the already budding punctuated evolution of how and why we do science in the larger sense. When we, the community, hold:

- "Science, Right or Wrong!"

we, as a *world* community, will *all* be far more likely to insist on:

- "when right, to be *kept* right;
  when wrong, to be *put* right."

We will find ourselves less inclined to accept or to teach science jingoism or chauvinism to our children, just as we (well, most of us, hopefully) today find ourselves *less* inclined to accept or to preach religious jingoism or chauvinism. We

will find that *any* hypocrisy will make it impossible for *anyone* to win the current "Science Wars"—other than Pyrrhically.

- **We will start a community "affirmative action" effort to seek out and *publicize* discovery of "… oversights" *just* as whole-heartedly as we seek out and publicize discovery of "… truth" (well, inspiring approximations to it). We will become far less accepting of science as a basis for or a means of the control of our daily lives, so anticonstitutional and so antithetical to our personal salvation—our personal freedom—that the Bible teaches us to sing of as the highest, greatest and most sacred good.**

Science—*and* mathematics—still have many more undiscovered… oversights. Discovery of these… oversights, and more importantly their careful examination and the soul searching that will go with it, need not make us sadder, but will help make us—both science and society—wiser. If we don't keep critically questioning accepted scientific beliefs, science will literally die out, far more quickly and completely than the dinosaurs or the passenger pigeons, and perhaps take the whole world with it.

7.14 "What is Reality?!" "What is Science?!" "What is Truth?!"

"Reality?!", an ancient question, with its especially famous sibling "Truth?!", and with "Science?!" trying to evolve into *the*—one and only?¡—bridge between them…

We don't have to be scientists to know that we see in/with only a small part of the at least 2-dimensional "visible light spectrum". (We usually just think of light as a single dimension of frequency/wavelength within which there is a band of "visible" frequencies/wavelengths, and within which color blindness is a serious possibility. But we also see only in/with only a limited dynamic range of the amplitudes of the visible frequencies/wavelengths, giving us another dimension. When we allow more than a single frequency/wavelength at a time, we can increase the dimensionality almost indefinitely.) We also have similar limitations for all of our sense-perception modalities, of which we normally think we have 5: sight, hearing, smelling, taste, and touch. (Buddhism speaks of a sixth sense that they call "mind", perhaps related to the ancient philosophers speaking of "knowing" as a sense-perception and sense-perceptive activity as opposed to our idea of "having stored accumulated knowledge".)

At present we humans have little sense of how many other sense-perception modalities are possible, or perhaps likely only a little ahead along our current evolution trajectories. So, in addition to the obviously great limitations of the 5/6 sense-perceptory modalities that we are somewhat familiar with, it is similarly obvious that we can only directly sense-perceive a tiny fraction of the reality we live in, surrounding us, not just off in the "infinite" cosmos somewhere. And within that perception, we have further limitations on what we can "conceive", the activity that turns sense-perceptions into information/intelligence.

Then we can add to this the ancient Hindu concept of "maya", the idea that we (normally) only perceive/conceive an "illusion", an "illusory world", and that the "underlying reality" is quite different than we either "perceive" or "conceive". (Maya reminds me of the famous Will Rogers quote: "It ain't what we don't know that gives us trouble, it's what we know that ain't so.")

So we start with a world that is an "illusory" perception/conception of a "infinitesimal" fraction of reality, and then we start to do our science. This involves abstractions that have unknown "fuzzy chaos coefficients". It also involves our further imposing inescapable approximations, further limiting our perception/conception of reality, again with unknown "fuzzy chaos coefficients". We can't be sure that our

"science" is helping us transcend our "maya", helping us find "detachment" from it. We can't be sure that it is not helping embed us even more in it, increasing our "attachment" to it.

And along with the sneaky chaos-type problems we have in knowing reality, we need to acknowledge those (misnamed) "uncertainty principle"-type considerations. When we try to "know" something more "completely", we use more "observations". These "observations" need energy to function and will "perturb" what we are trying to observe so that certain reciprocal aspects—of the abstractions we are trying to know most completely—are both made more "uncertain", and very possibly given a large percentage of the energy we are using to make the observations. E.g. when trying to ascertain the position of a particle with "perfect" accuracy, we will use a *lot* of energy in the process of observing it. We know that the uncertainty of the momentum of the particle goes to infinity as the position is measured to complete certainty. But we often ignore that the energy thus used to observe can *increase* the absolute *magnitude* of the momentum itself of the particle indefinitely, as well as increasing indefinitely the *uncertainty* of what that momentum is. (A variant of this shows up in social systems where the energy we use to try to bring about a certain "order" can add so much energy to the system that we instead get a resulting explosion of "disorder". "Order", it turns out, is a completely subjective concept.)

One of the reasons the search for "truth" has been so difficult can be described somewhat easily by referring to the above abstract rendering of the long distance and very indirect relationship between "ultimate reality" and our "scientific facts". We have had some important successes with abstractions—especially quantifiable and measureable—and reasoning with them, so many of us have left off evolving other ways of knowing "reality" and interacting with it. Our logic, our science, our "knowledge" in general, are all based on ever more abstract relationships among ever more abstract entities, with "chaos" and "uncertainty" thrown in for good measure. In our philosophical search for "truth" (as opposed to e.g. much earlier religious searches for truth) we have, starting with logic, evolved toward ever more abstraction (which of necessity is effectively a highly lossy data/information compression), toward ever greater quantification and associated "measurement". (Only "*measurable*" things are "*real*"?! This is Science at its most Pseudo.)

If, like our early astronauts used to have, you have a freeze-dried/dehydrated/-desiccated paste, you have certain expectations, hopes and prayers associated with it. You expect rehydration to make it possible to gag it down, you hope they didn't "abstract" out any essential nutrition, and you pray that some day they will make it truly edible. With our science and our dependence on abstraction, we are not as wise. When we use abstraction, we need to keep all such issues in mind. We can know that abstraction will mark out *part* of the "skeleton", *some* of the "bones" of the "truth", but we should also know that we need to "rehydrate" that with "flesh", and rehydrate that abstraction with "muscles", "ligaments", "nerves", "organs", etc.

Is a "map" of a "territory" even eligible to be considered to be "true" or "truth"?! If not, then how can a "statement" be "true"?! A "statement" might be only a very small part of a "map". Heraclitus reminds us that the river and man could easily not be the same twice faster than the map of the river might remind us of this, especially since we have a lot of inertia involved in updating maps, Earth observation and reconnaissance satellites to the contrary somewhat withstanding. And what about Gondwanaland? Uhh… Gondwana?! Reality, the territory, is an ever moving target. And even if it were standing still, it would be different from every point of view, and it would take us more or less forever to know even the static territory well enough to call

ourselves "omniscient". And the reality that we blindly attribute to our territory isn't static; it's very dynamic, much more than our current thought of it is, even if thought is quicker than the wind.

- ✓ **A gedanken experiment:** If, as the man on one bank of the river—here maybe sea is better—has one foot in sea and one on shore", and on the other side there is a woman likewise with "one foot in sea and one on shore", it doesn't take Einstein's special relativity to clue us into the fact that Heraclitus understated his case, since it's never even really the same river—or sea—even *once*…

  Our term "thing" really has no precise analog in reality, although we might suppose it has a chaotic-fractal approximate *abstraction*… or did have a second ago… Our term "cause" is even iffier, since it is only "every" "thing", somehow taken "together", that can "cause" "every" "thing".

  So we can infer from all this that the problem of $n = 2$ people talking even approximately about the same approximate "things" and "causes" is actually one of the most difficult problems in physics, as well as in life in general, getting more difficult as n increases. Relativity only begins to hint at these difficulties. Far from being the only valid way of "knowing" "reality", we are lucky that "science" can even "sometimes" give us an "abstraction" of the "truth", let alone a "sometimes" workable "approximation". We can almost hear Jake reminding us "A Science is a sometime thing, yes,…"

The problem with the search for "truth" is that we thought we needed to perfect our "maps (abstractions)" and our "map-making (abstraction-making) skills" rather than perfect our "knowing" the "territory" ("*un*-abstract whole truth"). Remember, to the ancients "to know" was a sense-perception (modality). It was present tense active rather than past tense accomplished (like "book learnin'"). A "map" needs someone who knows how to read it for it to have "true" value. And even then it will have its shortcomings, like not showing that the river had a white water rapids in a certain place. "Statements" may seem to be "true" or "false", but that is ultimately only in terms of a complex context, involving time, place, circumstances, people present, purposes (often cross), etc. So our efforts to find the perfect way to always make "all true statements"—and "only true statements"—have been fruitful only as a learning exercise… of how not to do "truth". But, "maps" *can* be very useful! even if they are not "true".

It is *only* "reality", both the little we perceive, even if more and more, and the far greater, even if less and less, that will always await our perception, that can possibly be the "truth" in any "ultimate" sense. That is the "truth" that we will "know", the "truth" that will "make us free"... or so we have been told. When we think of a "statement" as "true", a "scientific law" or a "scientific fact" as "true", whatever, they are "true by association", "true by ignorance", "true by lack of imagination", but they will never be the "reality" itself, they will never be "the truth", however useful they may be for a moment. Maps are means to know the territory of reality, not the knowing itself. At best they will help orient us toward perceiving—"*knowing*"—the "truth" of "reality". It was mentioned earlier that "Logic can best show us some of *how not* to reason, *not how* to reason." (See p. 100) But ultimately it is not our logic that must take the responsibility of making the leap from finite abstraction to the infinite *whole* truth of reality; it is *us*.

7.15 Finding A Lost Key

Another story [slightly modified by me] from another Idries Shah book tells us of the Mulla Nasrudin, and gives us a clue as to the possibilities—inspiring and not so inspiring—for the future evolution of science (quoted by me here without permission, as of this writing; there are countless variants of it in the public domain, but I especially like Idries Shah's works in general, and this might help introduce you to them):

> Someone saw Nasrudin searching for something on the ground [at night, under a lone streetlamp].
>
> "What have you lost, Mulla?" he asked. "My key," said the Mulla. So they both went down on their knees and looked for it.
>
> After a time the other man asked: "Where exactly did you drop it?"
>
> "In my own house."
>
> "Then why are you looking here?"
>
> "There is more light here than in my own house."
>
> Idries Shah, *The Exploits of the Incomparable Mulla Nasrudin*

Only an infinitesimal part of reality is sense-perceivable to us. Of that, only a very minute fraction is intelligible. Of that, only a tiny portion is qualifiable. Of that, only a very small percentage is quantifiable. Of that, not that much is actually measureable. Yet, quantifiability/quantification is where the light is for our modern science, and measurability is where our modern science finds this light to be adequate. What we scientifically measure becomes scientific fact, the only kind there is. This situation is insupportable. Our maps are at best dancing stick-figure models of reality, that can be very useful

The journey we just made so quickly above from reality to scientific fact can help give us a sense of how, in what direction, and how quickly, science needs to evolve, develop, mature, grow up… maybe by "becoming as a little child".

7.16 The Punctuated Evolution of "New" Science, Philosophy of Science, *and* "… Oversights"

Science should welcome "… when wrong, to be put right", but it Kuhnianly fails miserably in this, reacting… uh, "without fail" with some combination or other of "we won't even look *at* your telescope let alone *through* it" denial and, worse, Inquisitional anger. The first can be understandable if the real reason is that we are too busy looking at and/or through our own telescopes to look at and/or through the potential infinity of other people's telescopes. But that is not really denial in the sense we need to fear. It is denial and anger of the Inquisitional stripe that have no place of honor whatsoever in true science, in competent science.

If science wants to become other than that lone streetlamp helping people to find their keys, it must learn to come to terms with its "… oversights", learn to do "science… when wrong, to be put right." And this means more than just coming to terms with science's "Science Wars enemies", who also want "science… when wrong, to be put right", albeit in their own way. It also means coming to terms with actual serious failures of science on its own terms: science seriously—even egregiously—fundamentally fails *scientifically*, and will continue to do so until we learn to "put it right, and keep it right" on a "naturally" continuing basis. It means evolving science so that it is relevant to our own homes, not just scientifically in denial of their existence or significance.

It turns out, by the way, that not only are there many more such failures, but some of them are far more important than this albeit fascinating one, that I have been somewhat theatrically referring to as "Newton's Great… Oversight". Science must also learn to come to terms with its seemingly innate penchant for making these… oversights and taking sanctuary in denial, and in "self-righteous" anger, and in Inquisition, however refurbished. This penchant has been accepted by the community because of science's reputation, to date, a reputation that still does not even include truly self-acknowledging the possibility of making any serious… oversights.

Science *must* learn that it *is fundamentally fallible*, that it decidedly *can* fail, *on its own terms*, flagrantly, egregiously, even in its fundamental tenets, and even for centuries or millennia at a stretch, without noticing, and with *anger* when there are attempts to "question" it or to "put [it] right".

- ✓ NOTE: If you questioned that "millennia at a stretch", you have not been reading or thinking carefully. Aristotle was a modern scientist of his day, as was Ptolemy, and as were those who refused to look through Galileo's proffered telescope. "Ablative Shielding", described earlier, was used by the "modern science" of Galileo's and Newton's time to "disinherit" their by that time "scientific failures" and free themselves of associated reputations.

It is of telling significance that the term "… oversights" will be easier for scientists in the future to deal with psychologically than "blunders".

For Newton it can be considered a truly great "… oversight", but perhaps for other, lesser scientists it can be considered merely a normal "… oversight", an even more terrifying concept in its own way. In any case, dealing with this oversight—the non-zero falling rate difference of lighter and heavier bodies, along with its consequences—will be useful in this regard, since it is *so* obvious that *no* scientist since the time of Newton should have missed it. It is easy to demonstrate both the falling rate difference and its fascinating consequences in popular terms, without abstruse-arcane or even difficult mathematics. This will ease the way for the inevitable meta-paradigm shift to take place, perhaps in less than a Kuhnian generation! We will not merely wonder what Newton and Lagrange, and all the rest of us, may have missed. Actively and with integrity, especially with scientific integrity which has been known to lapse all too frequently, we will as a community start to investigate, discover and make public such… oversights Heeding this "wake up call" would indeed mark a definite "paradigm shift" toward transmuting our… *oversights* into… *insights*.

- "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

  Isaac Newton

  attribution, *Brewster's Memoirs of Newton*, Vol. ii, Chap. Xxvii (Brewster, David, Sir, 1781-1868, *Memoirs of the Life, Writings, and Discoveries of Sir Isaac Newton*, 1855)

- "I do not say that John or Jonathan will realize all this;
  but such is the character of that morrow which mere
  lapse of time can never make to dawn.
  The light which puts out our eyes is darkness to us.
  Only that day dawns to which we are awake.
  There is more day to dawn.
  The sun is but a morning star."

  Henry David Thoreau (1817-1862),
  the closing words of ***Walden; or, Life in the Woods***

[This page intentionally left blank.]

8 APPENDIX

Lagrangian Points L1-L5, Trojan Points, and Tadpole and Horseshoe Orbits

- ✓ **WARNING NOTE:** (see **Figure 4: The Lagrangian Points L1-L5, p. 139**) if you have spent much time searching the Internet for information on Lagrangian points, you have probably been confused by the lack of consistency in the numbering of the points. About the only consistency that you will find there is that there is agreement that L1-L3 are the collinear points, and that L4 is the leading equilateral triangle point, L5 the following. Sigh… If it helps, the numbering used here is the same as that found in e.g. the *Encyclopedia of the Solar System*, Weissman, McFadden, Johnson, Eds.; Academic Press, 1999, pp. 815-7.

Using his perturbation theory (a major extension of the calculus of variations and partial differential equations), Lagrange found 5 points—referred to collectively as the "**LAGRANGIAN POINTS**"—where an "infinitesimal" body would theoretically maintain its position relative to the 2 non-infinitesimal bodies as they all move through space (only if completely unperturbed, especially in the case of **L1**, **L2**, and **L3**; these last 3 are considered positions of "unstable equilibrium"). These are Lagrange's "homographic solutions" to the equations of motion.

A "**HOMOGRAPHIC SOLUTION**" is (here) a solution of the equations of motion that retains the same shape—i.e. geometrically similar, but not necessarily the same size or orientation—as the system evolves through time. I.e. the ratios of the distances between each pair of points remain the same. In particular, for the (only) solutions involving non-collinear points, the equilateral triangle formed by the 3 points remains geometrically similar to its initial configuration; i.e. it remains an equilateral triangle as it rotates, expands and contracts.

(Warning: the terms "homographic" and "homography" frequently have other scientific and/or mathematical meanings.)

Three of these solutions, **L1**, **L2**, and **L3** are collinear—i.e. all 3 bodies lie on a straight line as they move through space, revolving around their common center of gravity. The other two, **L4** and **L5**, are the **TROJAN POINTS** at the equilateral triangle positions (which we have just found by questioning Galileo's hypothesis that lighter and heavier bodies fall at the same rate). Asteroids that orbit these points are known as "**TROJAN ASTEROIDS**". Jupiter's Trojan asteroids can take hundreds of years to complete such an orbit. (They are mostly in "tadpole" orbits, but probably at least some are in "horseshoe" orbits. See below.)

Although **L1-L3** are theoretically stable if "unperturbed" (this is the meaning of the homographic solution result), they are known to be unstable in fact (or thought to be; see below). Infinitesimal bodies placed there will eventually "wander away" under the influence of "perturbations" induced by e.g. the gravitational influence of other planets.

(See **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**.) The points **L4** and **L5** are considered "stable" (see **Section 3.12, Stability?!, p. 62**). I.e. if the perturbations are small enough, infinitesimal bodies placed there will stay near the Trojan points in relatively stable "**TADPOLE ORBITS**", elongated, non-elliptical, asymmetrically curved orbits. If the perturbations on Trojan bodies are large enough and concerted enough, their tadpole orbits might even "grow" in such a way that

(mixing metaphors) the **L4** and **L5** tadpoles meet and form still "stable" "**HORSESHOE ORBITS**", as seen in the reference frame of e.g. the Earth. The more energy the 3rd body has, the larger the tadpole or horseshoe orbit will be, and the easier it might be to perturb it sufficiently for it to escape. **Figure 5**, **p. 140**, shows these "concentric" tadpole and horseshoe orbits. Asteroid 3753 CRUITHNE is an example of an "Earth companion", a body in a very peculiar horseshoe orbit relative to Earth. As of this writing, two of the web sites that give interesting info about Cruithne are:

- http://www.astro.uwo.ca/~wiegert/etrojans/etrojans.html and
- http://focus.aps.org/story/v4/st16

L1 has become famous because that is where **SOHO**, the SOlar and Heliospheric Observatory, is stationed. Due primarily to the Earth-Sun mass ratio, **L1** is about 1% closer to the Sun than the Earth, so about 930,000 miles sunward from Earth. (**L2** is about 1% further away from the Sun.) The lack of stability means that it must use fuel to keep itself sufficiently near **L1** for solar observation. Actually it orbits **L1** in what is called a "Halo Orbit". (NASA has web pages that give interesting details.) This special orbit actually keeps it somewhat *away* from **L1**, which is in direct line of sight with the Sun, i.e. enough away from solar interference to send data back. Roughly every 27 days SOHO must readjust its orbit.

L3 is almost precisely the same distance away from the Sun as Earth (approximately millionths of a percent difference), and on the opposite side. This is because the gravitational effect of Earth is very small at twice its distance to the Sun. **L3** is the place where science fiction has placed some of its alternate Earths, but scientists think they are *sure* that this is not feasible because, as with **SOHO** in **L1**, its instability means that a body (at least an "infinitesimal" body) would drift away from **L3** if energy were not expended to keep it there. But let us remind ourselves: scientists are *sure* about this in the same way they have been *sure* for over 300++ years that lighter and heavier bodies fall at precisely the same rate!

L3 Stability (2): By the way, you may have noted (see **Figure 5: "Tadpole" and "Horseshoe" Orbits, p. 140**) that the **L3** point is in the "band" of the "concentric" horseshoe orbits, and that this might technically make it a point of "stable" equilibrium since the tadpole orbits are considered part of the stability of **L4** and **L5**, and they expand and merge into the horseshoe orbits. This also shows a way toward a significant advance in the evolution of our concept of stability! Good for you! (See **L3 Stability (1)** on **p. 64**.) Science—and mathematics—need to have their inconsistencies pointed out. (Now, what about **L1** and **L2**?!…)

9 REFERENCES AND BIBLIOGRAPHY

9.1 Books

Brewster, David, Sir, *Memoirs Of The Life, Writings And Discoveries Of Sir Isaac Newton*, Vols. 1 and 2, 1855, Kessinger Publishing, LLC, 2004
Reference: *Vol. ii, Chap. Xxvii*
(some people think of this as less of a biography and more as a massive post-Renaissance prequel to our post-modern image management)

Elliot, T. S., *Four Quartets*, 1935-42, *Little Gidding* (the 4th quartet), 1942
Reference: towards the end of its section 5.

Encyclopedia of the Solar System; Weissman, McFadden, Johnson, Eds.; Academic Press, 1999; 2nd edition, McFadden, Weissman, Johnson, Eds., 2006;
Reference: pp. 815-7 (of 1999 ed.)

Fuller, Steve, *Thomas Kuhn: A Philosophical History for Our Times*, University of Chicago Press, 2000

Fuller, Steve, *Kuhn vs. Popper, the Struggle for the Soul of Science*, Columbia University Press, 2004
Reference: p. 129 ("… bewildering tendency to retreat…"), from p. 120

Fuller, Steve, *Science vs Religion? Intelligent Design and the Problem of Evolution*, Polity, 2007

Galilei, Galileo, *Dialogue Concerning the Two Chief Systems of the World* (*Dialogo sopra i due massimi sistemi del mondo*), 1632

Gould, Stephen Jay, *The Structure of Evolutionary Theory*, Cambridge, MA: Belknap Press of Harvard University Press, 2002

The Handbook of Science and Technology Studies, 3rd Ed., Hackett, Amsterdamska, Lynch, Wajcman, Eds., Cambridge, MA: MIT Press, 2007
(together with the third edition, the second edition, which can still be found, helps give a historical picture of the evolution of STS; I have not succeeded in finding the first edition)

Hirshfeld, Alan, *The Electric Life of Michael Faraday*, Walker and Company Publishing, 2006
Reference: Ampère's remark to a friend on Oersted's discovery, pp. 73-4.

Kuhn, Thomas, *The Structure of Scientific Revolutions*, 1962, 1970, 1996
Reference: p. 62. ("… the consequent change of paradigm categories and procedures often accompanied by resistance…"), from p. 114
(BTW, Fuller holds that the 1st edition of 1962 is significantly different from the 2nd of 1970, and he may be right; I am trying to compare editions as I write this; the third edition is probably much like the 2nd in this regard)

Losee, John, *A Historical Introduction to the Philosophy of Science*, first pub. 1972, New York: Oxford University Press, 4th Ed., 2001
Reference: p. 1 ("Unfortunately, philosophers and scientists are not in agreement on the nature of the philosophy of science."), from p. 80.

Michener, James A., *Centennial*, first published in 1974
Reference: somewhere in there, if I am remembering the right Michener book ("sod crop"), from p. 16

Nahin, Paul J., *Oliver Heaviside, The Life, Work and Times of an Electrical Genius of the Victorian Age*, 1988, Johns Hopkins University Press, edition with new Preface, 2002

Newton, Isaac, *Opticks*, first released to public in 1704; Dover, first edition, 1952, revised edition, 1979
Reference: see in general and in particular the Preface by I. Bernard Cohen (revised ed.), p. xi (for an example of the "Newtonian Inquisition" against Thomas Young in the very early 1800s), and in general the Introduction by E. T. Whittaker (first and revised eds.)

Newton, Isaac, *Philosophiae naturalis principia mathematica*, first published in 1687, a new translation by I. Bernard Cohen and Anne Whitman (assisted by Julia Budenz), preceded by "A Guide to Newton's *Principia*" by I. Bernard Cohen, University of California Press, 1999

Popper, Karl, *The Logic of Scientific Discovery*, first published in German in 1935 as *Logik der Forschung*, Verlag von Julius Springer; first published in Routledge Classics 2002 by Routledge

Raymo, Chet, *book review* in Scientific American (September, 2000) of Steve Fuller's then new book *Thomas Kuhn: A Philosophical History for Our Times*, University of Chicago Press, 2000

Schiller, F. C. S., "Scientific discovery and Logical Proof", in *Studies in the history and method of science*, C. Singer, Ed.
Reference: to §3, p. 237, from p. 97
Reference: to §7, pp. 240-2, from p. 100.)

Shah, Idries, *The Exploits of the Incomparable Mulla Nasrudin*, 1st paperback ed., E. P. Dutton, 1972
p. 26 ("There is more Light here"), from p. 125
(BTW, just about any book by Idries Shah is worth reading)

Thoreau, Henry David, *Walden*, 1854; many available.
Reference: last paragraph ("… the sun is but a morning star."), from p. 127.

Westfall, Richard S., *Never at Rest, A Biography of Isaac Newton*, Cambridge University Press, first published in 1980, first paperback edition in 1983, 14th printing in 2005
(this book gives us one of the most real pictures of "the real" Newton, both the man and the natural philosopher, his personal life, his scientific life, his alchemical life… knowing that he did a *lot* of alchemical mercury helps one understand the later part of his life a *lot* better… compare Westfall's work with the Newton "bio" by Brewster)

9.2 Internet Web Sites

Newton, the "last of the magicians"; John Maynard Keynes in his *Newton, the Man*
http://www-groups.dcs.st-and.ac.uk/~history/Extras/Keynes_Newton.html [link OK as of Jun 2008] (from p. 26)

"Simon Stevin, Flemish tutor of a Dutch Prince"
http://users.ugent.be/~gvdbergh/files/publatex/stevinoe.html [link OK as of Jan 2009]
Reference: very quick bio; Stevin actually dropped lighter and heavier bodies from a church tower in 1586, beating out Galileo by several years (from p. 30)

More of the fascinating history of the discovery of the Trojan asteroids
http://www.cfa.harvard.edu/iau/pressinfo/TheFirstTrojanObs.html [link OK as of Jun 2008]
(from p. 31)

For more on 3753 Cruithne
http://focus.aps.org/story/v4/st16
[link OK as of Jun 2008] (from p. 68 and p. 130)
and
http://prola.aps.org/abstract/PRL/v83/i13/p2506_1
[link OK as of Jun 2008] (from p. 68)
and
http://www.astro.uwo.ca/~wiegert/etrojans/etrojans.html
[link OK as of Sept 2008] (from p. 130)

The Near Earth Asteroid Rendevous mission
http://near.jhuapl.edu/
[link OK as of Jun 2008] (from p. 69)

The Jupiter Trojans
http://en.wikipedia.org/wiki/Trojan_asteroid
[link OK as of Jul 2008] (from p. 72) Wikipedia generally offers lots of good info.

The (Witkowski) Kordylewski Cloud(s), space dust near Earth's L4 and L5.
http://en.wikipedia.org/wiki/Kordylewski_cloud
[link OK as of Feb 2011] (from p. 69) Wikipedia generally offers lots of good info.

The Hildas (group of asteroids associated with Jupiter)
http://en.wikipedia.org/wiki/Hilda_family
[link OK as of Jan 2009] (from p. 64) Wikipedia generally offers lots of good info.
Reference: for the view of the ecliptic distribution, look near the end of the page.

Mann, Jewitt, and Lacerda, "Fraction of Contact Binary Trojan Asteroids", The Astronomical Journal, 134:1133–1144, 2007 September; abstract
http://www.iop.org/EJ/abstract/-search=56910649.1/1538-3881/134/3/1133
[link OK as of Jun 2008] (from p. 73)

Miles, Kathy, and Peters II, Charles F., "Seeing Further, The Legacy of Robert Hooke", 1996-2002
http://starryskies.com/articles/spec/hooks.html
[link OK as of Dec 2008] (from p. 23)
Reference: Newton's first run-in with Hooke over theories of light (from p. 88)

Title: A Search for Objects near the Earth-Moon Lagrangian Points
Authors: Valdes, Francisco and Freitas, Robert A., Jr.
Journal: *Icarus* **53**, 453-457 (1983)
http://www.rfreitas.com/Astro/SearchIcarus1983.htm [link OK as of Feb 2011]
Reference: **Section 4.3, Trojan Space Debris?! Trojan Atmospheres?!, p. 68**, and **p. 69** for an UPDATE giving a quick description of the still disputed "Kordelewski cloud" near one of the Earth-Moon Trojans.)

Title: A List of Objects at Lagrangian Points
http://en.wikipedia.org/wiki/List_of_objects_at_Lagrangian_points [link OK as of Nov 2011]
Reference: **Section 5.2, Trojan Point Astronomy in the 20th and Early 21st Century, p. 73**.

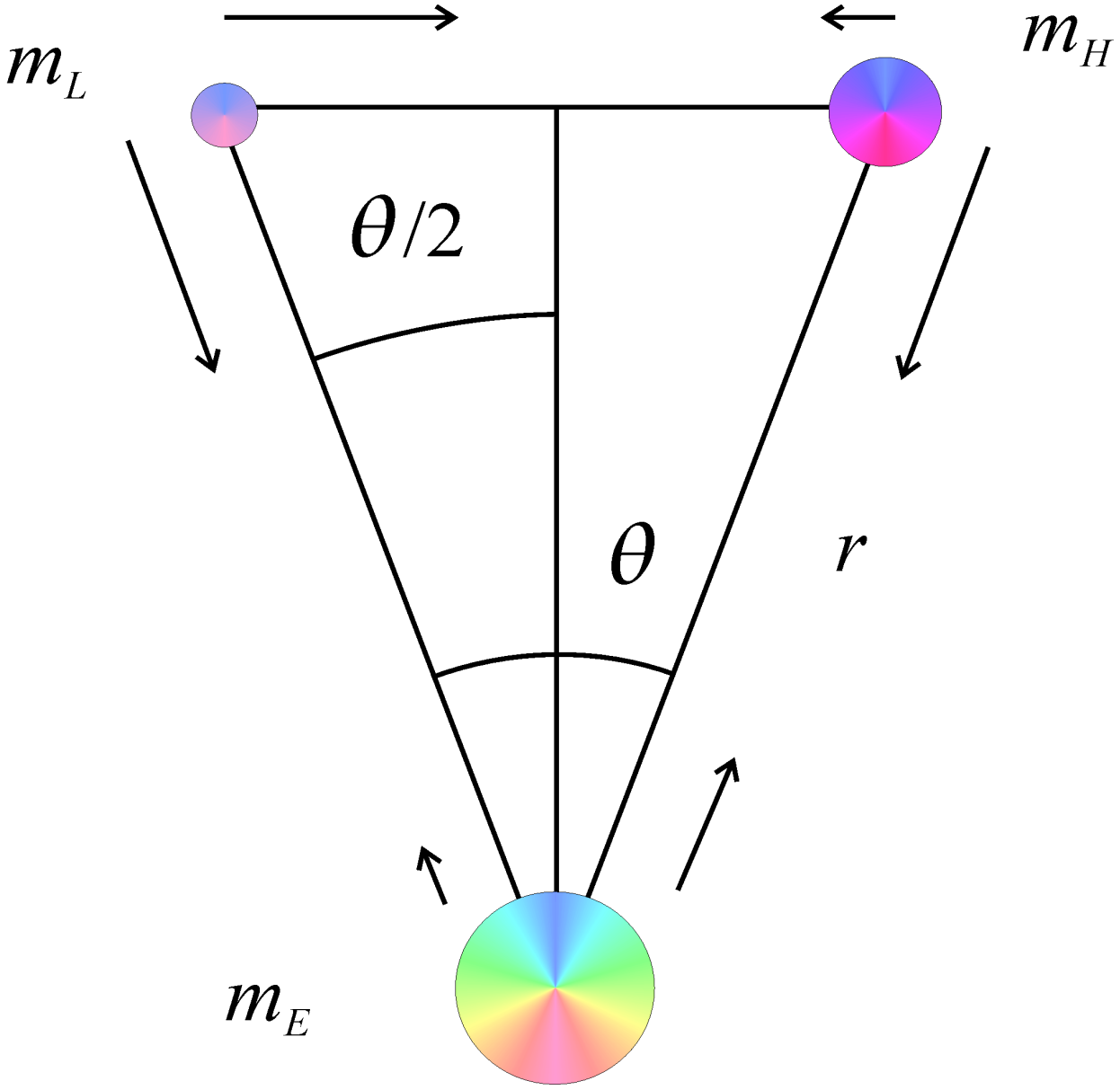
Title: Earth's Trojan Asteroid
Authors: Martin Connors, Paul Wiegert, and Christian Veillet
http://www.nature.com/nature/journal/v475/n7357/full/nature10233.html [link OK as of Nov 2011]
Reference: **Section 5.2, Trojan Point Astronomy in the 20th and Early 21st Century, p. 73**.

Title: A Family of Retrograde Orbits around the Triangular Equilibrium Points
Authors: Szebehely, Victor, and Flandern, T. Van
Journal: Astronomical Journal, Vol. 72, p. 373 (1967)
Bibliographic Code: 1967AJ.....72..373S
http://adsabs.harvard.edu/full/1972IAUS...44..353D [link OK as of Mar 2010]
Reference: Figure 6a: Are Stable Trojan Star Systems Possible?! Part a (from p. 142.)

Title: The Velocity-Distance Relation and the Hubble Constant for Nearby Groups of Galaxies
Author: de Vaucouleurs, Gérard Henri
Journal: External Galaxies and Quasi-Stellar Objects, Proceedings from IAU Symposium no. 44 held in Uppsala, Sweden, 10-14 August 1970. Edited by David Stanley Evans, Derek Wills, and Beverly J. Wills. International Astronomical Union. Symposium no. 44, Dordrecht, Reidel, p.353
Bibliographic Code: 1972IAUS...44..353D
http://adsabs.harvard.edu/full/1972IAUS...44..353D [link OK as of Dec 2008]
Reference: "hierarchical" star systems (from p. 66.)

10 FIGURES

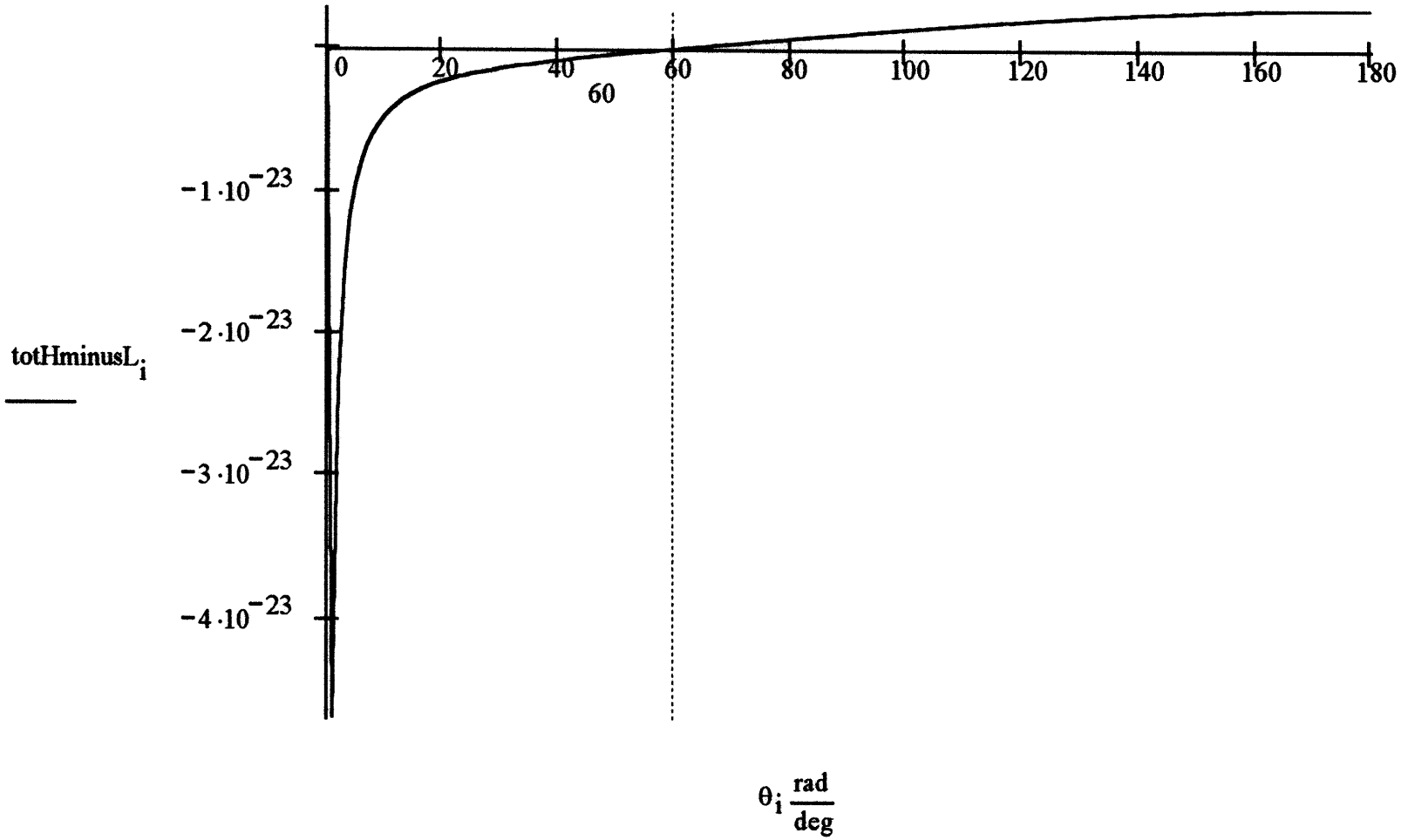
- ✓ **GENERAL NOTE: all these Figures—diagrams and plots—are for the *static* part of the analysis, which does not take into account the 3-D dynamics of the Trojan bodies, e.g. of revolution around/orbiting their common center of mass or the effects of perturbations.**



10.1 Figure 1: The 3 Bodies/Masses, and the Angles and Distances Among Them

Each body has acceleration components due to the other 2 masses (not to scale, of course).

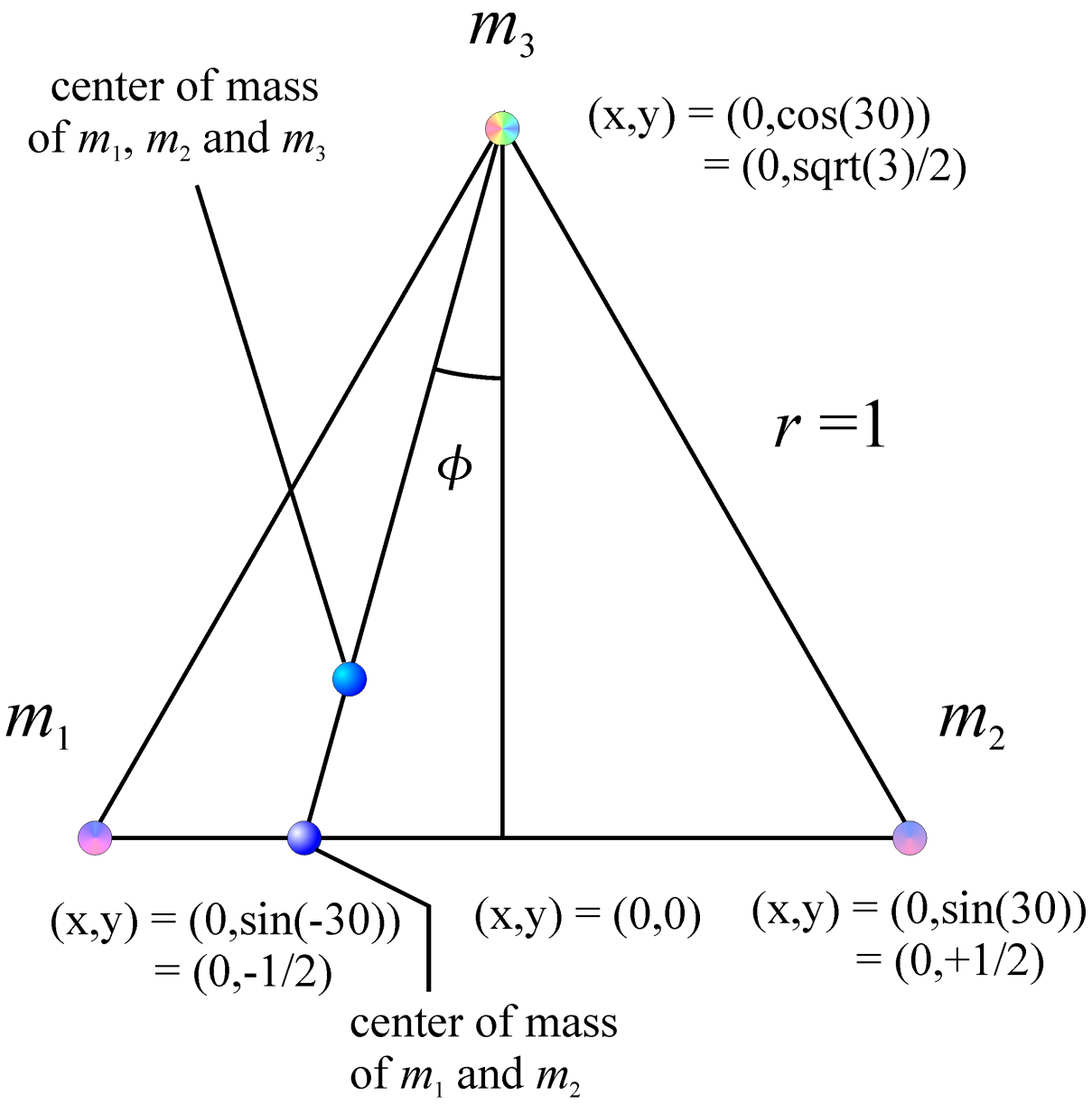
✓ NOTE: the accelerations of the 2 bodies (m_L and m_H) toward each other have non-zero components in the direction of the center of the Earth (m_E).



10.2 Figure 2: The Difference in Falling Rates as a Function of Angular Separation

The plot is in degrees (the numbers across the top).

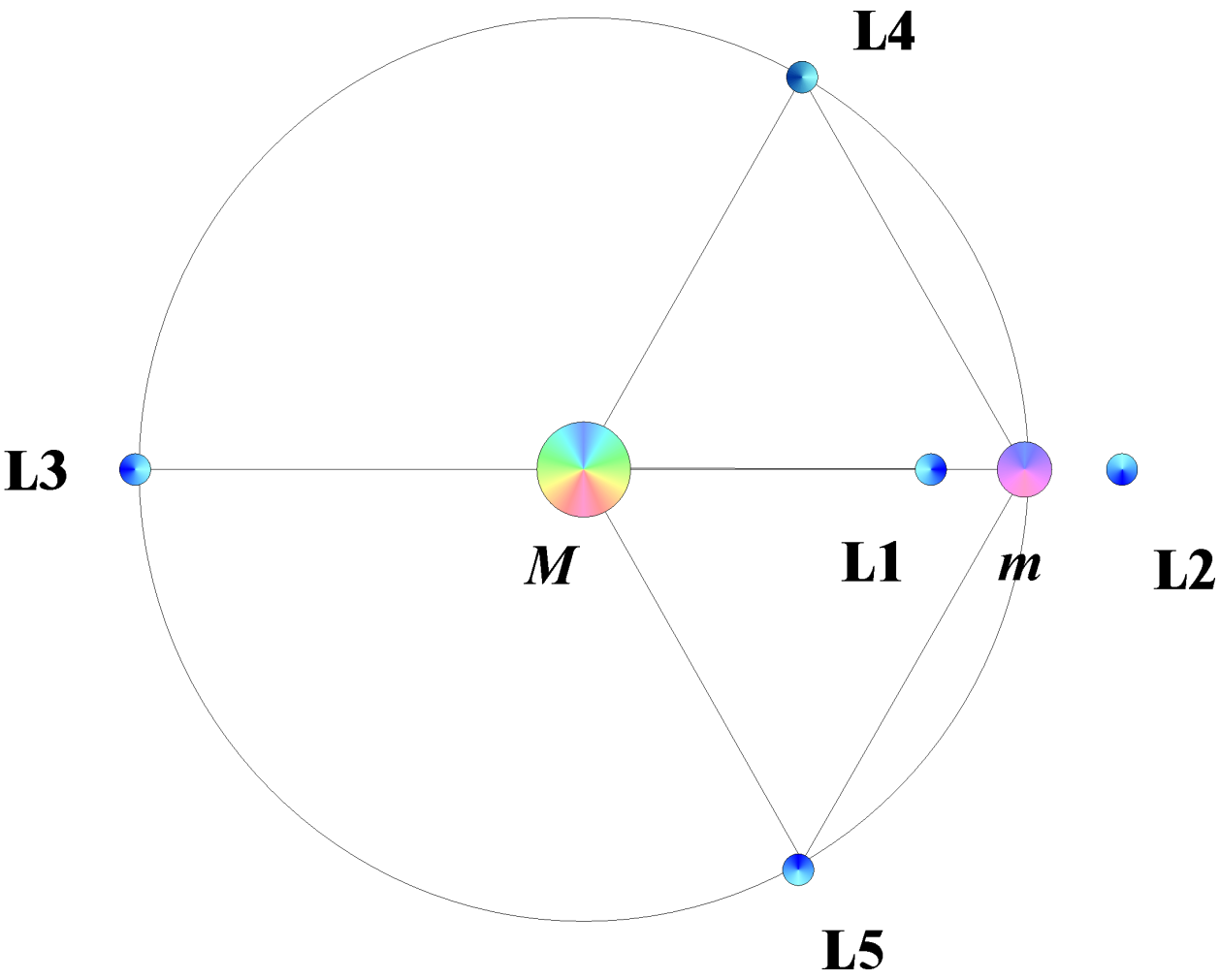
- ✓ NOTE: it zeroes at 60 degrees, when all 3 bodies are at the vertices of an equilateral triangle.
- ✓ NOTE ALSO: the spike upward (hard to see) at 0 degrees is the MathCAD 2000 false 0 for the singularity at 0 degrees.



10.3 Figure 3: The 3 Masses and 2 of Their Centers of Mass

The 3 masses at the vertices of an equilateral triangle, with each side a unit distance in length, and with both the masses m_1 and m_2 positioned on the x-axis ($y = 0$) and equidistant from the y-axis ($x = 0$).

NOTA MOLTO BENE: STABILITY?! As this diagram shows, the alignment of the top mass (m_1) with the center of mass of all 3 and the center of mass of the other 2 masses (in that order) holds for *all 3 masses*. These alignments are essential to maintaining the static equilateral triangle sans perturbation(s). The question is what happens when they are all rotating around their common center of mass and the configuration is perturbed "slightly" away from the equilateral triangle. There are lots of ways this can happen, and that's why perturbation theory, arcane as it is, gives a more satisfyingly complete understanding of this than we have come up with so far with just algebra and trig.



10.4 Figure 4: The Lagrangian Points L1-L5

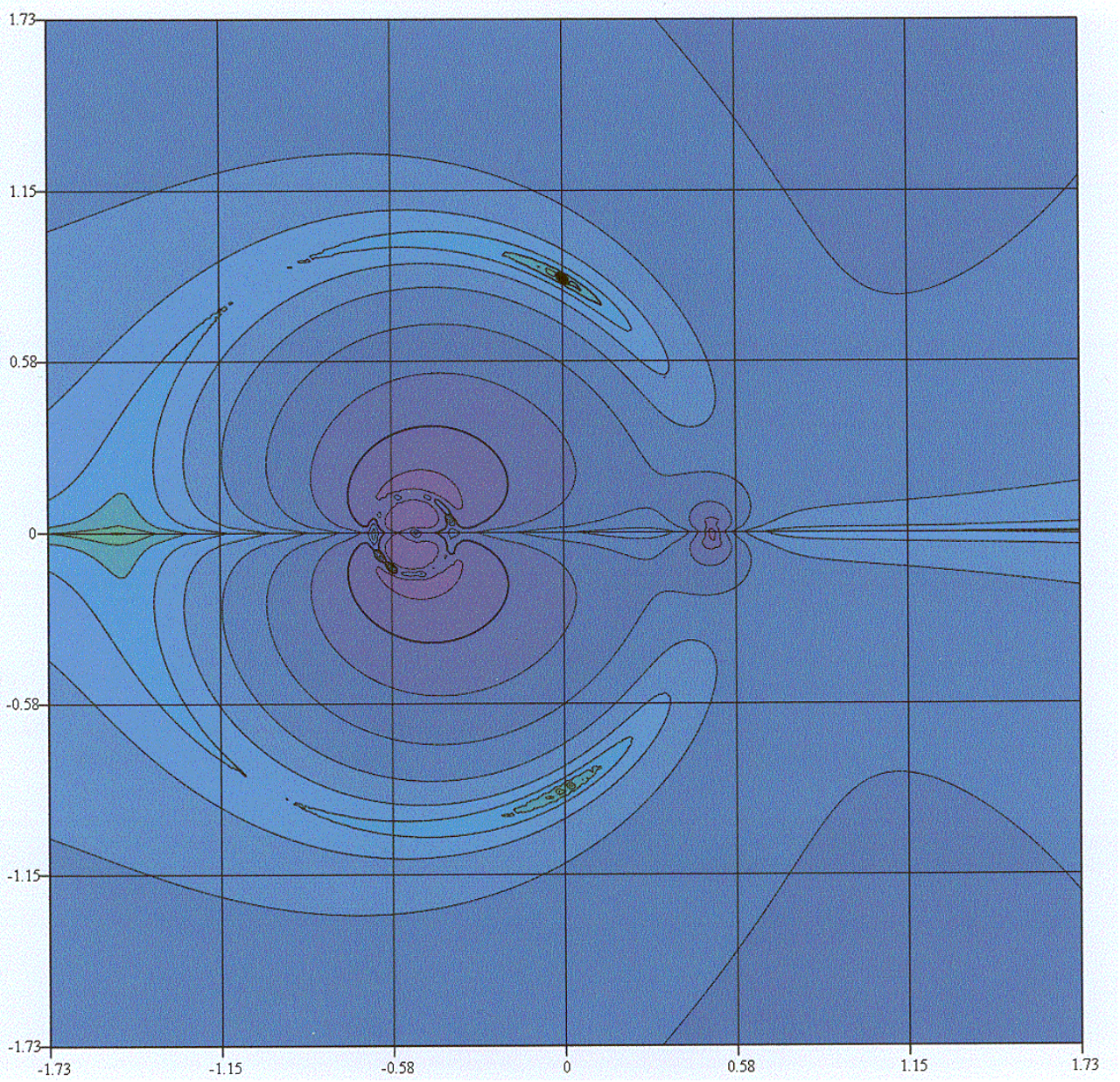
Lagrange's theory assumes that **L1-L5** are occupied only by "infinitesimal" bodies (purely a calculational convenience, since no masses are truly "infinitesimal"). Here the diagram assumes that $m << M$.

✓ NOTE: technically, the Lagrangian points **L4** and **L5** do not even exist unless Lagrange's restriction is met, i.e. that $m < \sim 0.04\ M$.

The relative positions of **L1** and **L2** (and even **L3**, but far less so) with respect to the two non-infinitesimal masses depend on the mass ratio, m/M, but **L4** and **L5** always form equilateral triangles with m and M.

✓ NOTE: it is implicit in the above diagram that mass m is *much* smaller than mass M, i.e. the center of mass of both masses together is very close to the center of mass of M itself. This is the case with the Sun and Jupiter. However, the more general case needs to be studied, and can be with the aid of computers.

✓ ALSO NOTE: if you have been searching the Internet for info on Lagrangian points, you have probably been confused by the lack of consistency in the numbering of the points. About the only consistency that you will find there is agreement that **L1-L3** are the collinear points, and **L4-L5** are the equilateral triangle points.



10.5 Figure 5: "Tadpole" and "Horseshoe" Orbits

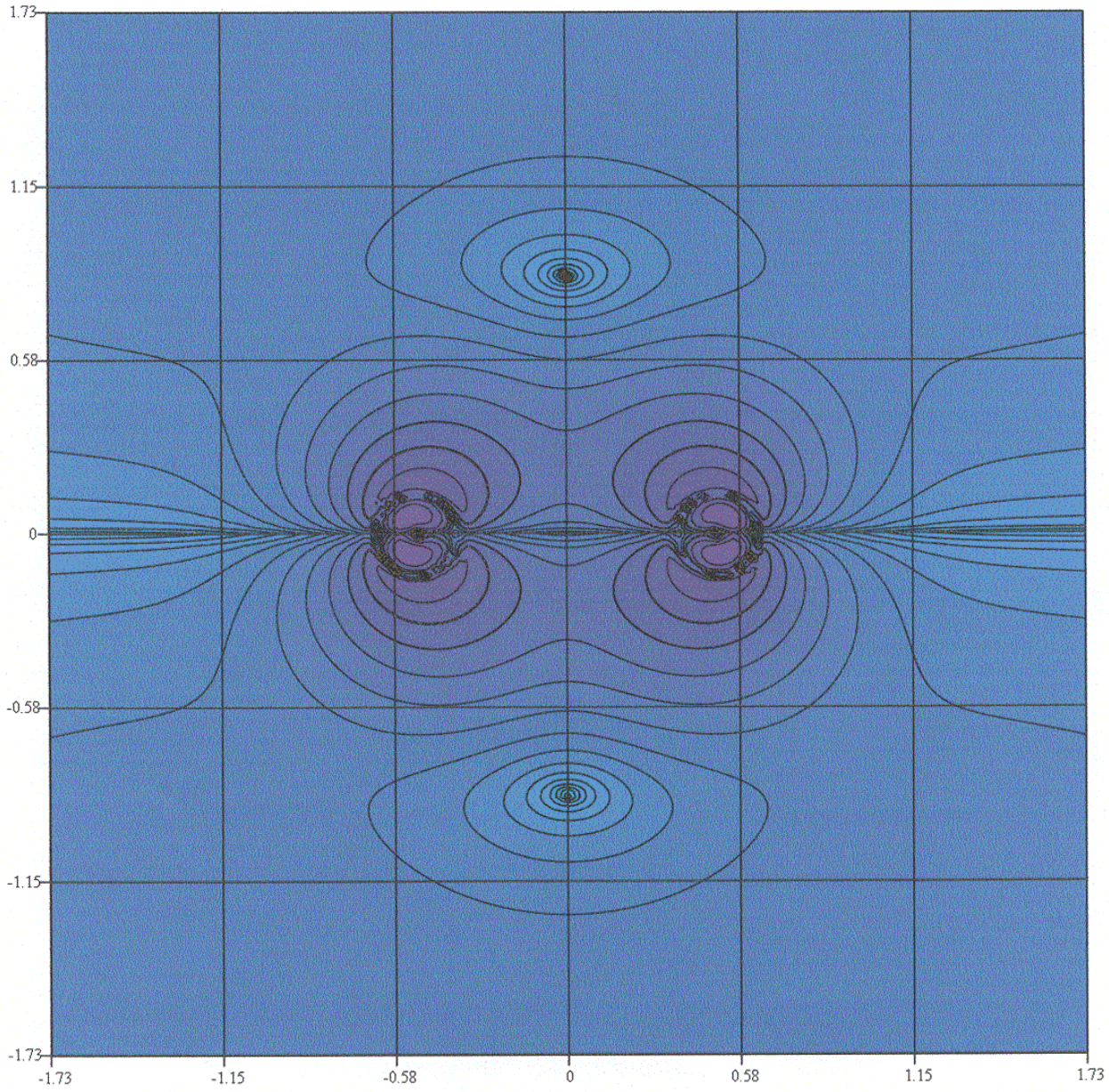
This contour plot gives a good idea of what "tadpole" and "horseshoe" orbits are all about. It plots a function of the falling rate difference that indicates the rate at which a (static, i.e. no initially non-zero velocities) triangle of masses, with the "infinitesimal" body at **L4** (or **L5**) perturbed (to the point on the plot), further degrades from equilateral. The degradation is least quick in the tadpoles, quicker in the horseshoes, to very much quicker outside them. The largest mass is set to "1"; the second largest mass is 0.01 m_1, i.e. smaller than the ~ 0.04 m_1 that Lagrange's analysis indicated was the upper level for m_2 that allowed stability (reminder: with the 3rd mass "infinitesimal"); the smallest mass is 0.0001 m_1, and so approximates "infinitesimal".

The "centers" of the mirror-symmetric tadpoles—**L4** and **L5**—lie on the vertical $x = 0$ line, roughly 0.866 above and below the y = 0 line.

L1, **L2** and **L3** all lie on the horizontal $y = 0$ line. On this plot, **L1** is about $x = 0.3$, L2 about $x = 0.9$, and **L3** about $x = -1.5$.Regarding **L3**, the more equal the 2 smaller masses are, the more likely that they—one of them at **L3**—will be the same distance from the largest mass.

- ✓ NOTE: how the tadpoles "grow" till they meet and form an oddly shaped horseshoe, one which covers **L3**. **L3** is actually in a place where the (degenerate) triangle degrades roughly as fast as it does in the tadpoles. This suggests that **L3** should be thought of as stable if the horseshoe orbits are considered stable!… unless e.g. the velocity through the **L3** region is critical to its returning, and could not generally be obtained by "wandering away, perturbed" from **L3**.
- ✓ NOTE ALSO: a contour plot is like a topographical map: the shape of the contour lines depends on the "elevation" of the intersections of various "cutting planes" with the function's 3-dimensional surface. The contour lines of the same surface can look quite different if contour-plotted slightly differently.
- ✓ NOTE: we think of the asteroid as being in a tadpole or horseshoe orbit with regard to e.g. the Earth or Jupiter, since we are looking at its orbit from an Earth or Jupiter relative reference frame. If we look at the orbit from a frame that is where Earth would be if there were no asteroid, and look at both the Earth and the asteroid orbiting, the Earth will also be seen to have its own "tadpole" or "horseshoe" type movement, but smaller as its mass is larger. The asteroid's orbit will also have a somewhat different shape. If the masses are more equal, the orbits can take on very different shapes. See examples of possible shapes in **Figure 6b**, **p. 144**, which has all 3 masses equal.
- ✓ ALSO NOTE: *technically*, the Lagrangian points would not even exist *if* Lagrange's restriction on relative mass, i.e. that $m_2 < \sim 0.04\ m_1$, is not met.
- ✓ FURTHER NOTE: this 2 dimensional plot is inadequate to give a picture of e.g. the dynamics of the Earth companion asteroid, 3753 Cruithne, with its highly inclined orbit that takes it directly *above* Earth.

(Plotted with MathCAD 2000.)



10.6 Figure 6a: Are Stable Trojan Star Systems Possible?! Part a

In this plot, the 2 "unperturbed" masses are equal, $m_1 = m_2$, and the 3rd, "perturbed" mass at **L4** (or **L5**) is again 0.0001 m_1, and so approximates "infinitesimal", and all 3 are in an equilateral triangle. (See **Figure 6b**, **p. 144**, for 3 equal masses.) According to Lagrange's analysis, this system should *not* be stable. As in **Figure 5**, **p. 140**, this contour plot shows a function of the falling rate difference that indicates the rate at which a (static) triangle, here of equal masses, with the body at **L4** (or **L5**) perturbed (to the point on the plot), further degrades from equilateral. The "tadpoles" no longer have that characteristic, asymmetric shape, but rather have "degenerated" to a seemingly simpler, symmetric shape, since the 2 "unperturbed" masses are equal. Again, the degradation is least quick in the degenerated "tadpoles", quicker in the degenerated "horseshoe" (which almost

seems to have formed a circular orbit around both masses; actually, see **Figure 6b**, **p. 144**, since it shows this much better), to very much quicker outside them.

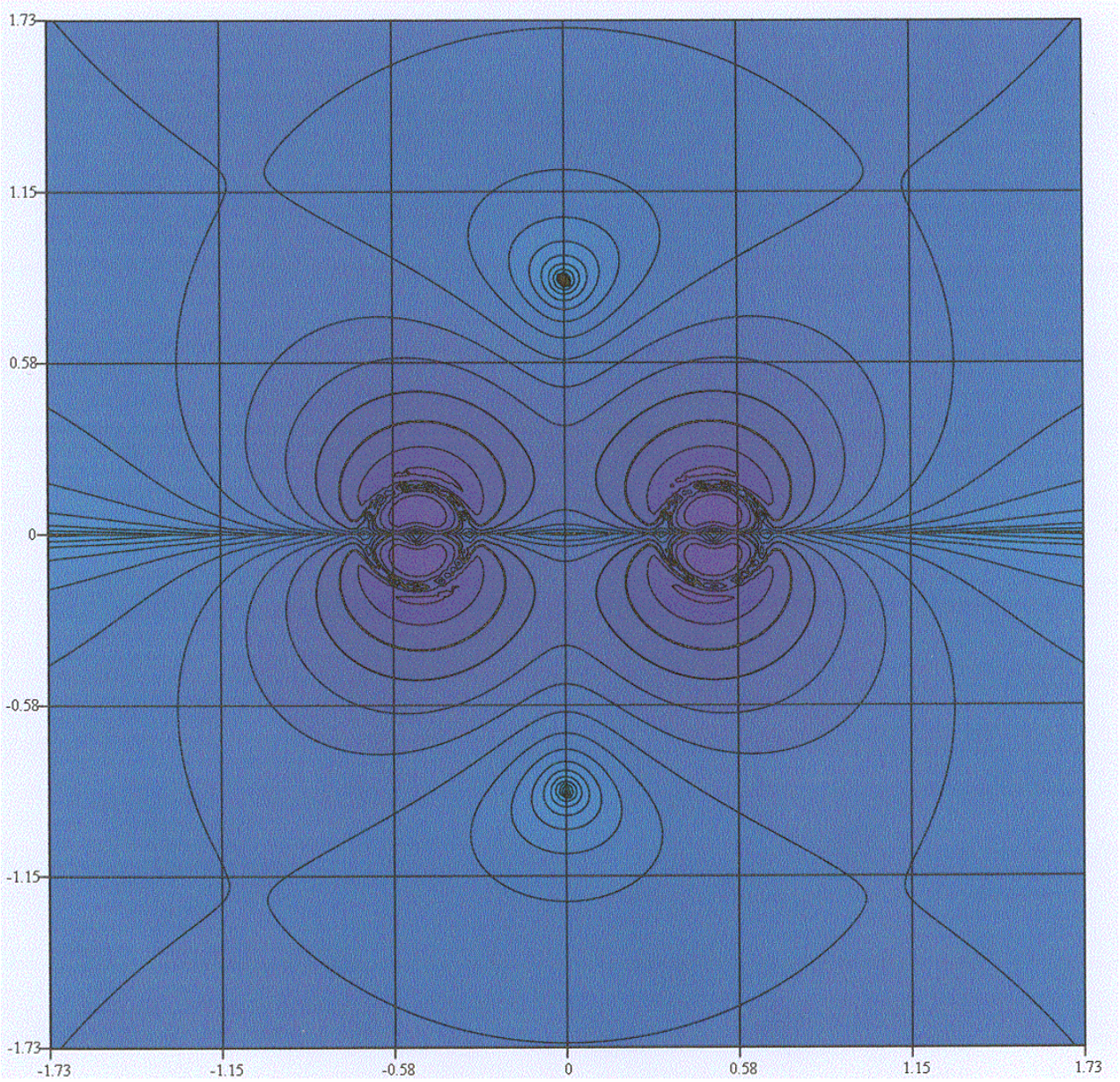
It may be difficult to tell from this plot, but the region of possible stable "equilibrium" around **L4** (and **L5**) seems larger in this plot with an "infinitesimal" 3rd body (i.e. the lighter color near **L4**/**L5** extends further out; the contour lines are almost irrelevant in this) than the comparable region in **Figure 6b**, **p. 144**, with all 3 masses equal. Comparing **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**, suggests that a wide range of Trojan-ternary star systems or binary star systems with a planet at **L4** or **L5** are possible.

- ✓ NOTE: **L1** has moved to a place of symmetry between L4 and L5, symmetrical since the 2 major masses are equal. **L2** and **L3** do not even appear on this plot as they are ~ 1.4 times the distance between the 2 major masses, but beyond on either side.
- ✓ NOTE ALSO: a contour plot is like a topographical map: the shape of the contour lines depends on the "elevation" of the intersections of various "cutting planes" with the function's 3-dimensional surface. The contour lines of the same surface can look quite different if contour-plotted slightly differently. Some of the difference between the contours of **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**, is merely that, some that the region of possible stability around the Trojan points is narrower when all 3 masses are equal.
- ✓ ALSO NOTE: technically, the Lagrangian points do not even exist in the case of 2 (or 3) equal masses, since Lagrange's restriction, i.e. that $m < \sim 0.04\ M$, is not met.

To examine stability carefully, it would be necessary to look at the total dynamics of the system, but computers should make that quite feasible, even without making the simplifying assumption—currently considered necessary—of "infinitesimality" for any of the masses. Even if it is unstable, a Trojan-ternary star system could take an astronomically significant time to degrade. And it's good to remember: the scientists who think ternary star systems must be unstable and must therefore be impossible also think lighter and heavier bodies always fall at precisely the same rate.

(Plotted with MathCAD 2000.)

- ❖ The reader might be interested in a 1967 article by Victor Szebehely and T. Van Flandern titled "A Family of Retrograde Orbits around the Triangular Equilibrium Points". It can be found at: http://articles.adsabs.harvard.edu//full/1967AJ.....72..373S/0000373.000.html It has neat plots of orbits of an "infinitesimal" body around two equal mass co-orbiting bodies.



10.7 Figure 6b: Are Stable Trojan Star Systems Possible?! Part b

In this plot all 3 masses are equal (see **Figure 6a**, **p. 142**, for 2 equal masses and an "infinitesimal" perturbed mass) and in an equilateral triangle. According to Lagrange's analysis, this system should *not* be stable. As in **Figure 5**, **p. 140**, and **Figure 6a**, **p. 142**, this contour plot shows a function of the falling rate difference that indicates the rate at which a (static) triangle, here of equal masses, with the body at **L4** (or **L5**) perturbed (to the point on the plot), further degrades from equilateral. The "tadpoles" no longer have that characteristic, asymmetric shape, but rather have "degenerated" to a seemingly simpler, symmetric shape, since the 2 "unperturbed" masses are equal. Again, the degradation is least quick in the degenerated "tadpoles", quicker in the degenerated "horseshoe" (which almost seems to have

formed a circular orbit around both masses), to very much quicker outside them.

It may be difficult to tell from this plot, but the region of *possible* stable "equilibrium" around **L4** (and **L5**) seems smaller in this plot with all 3 masses equal (i.e. the lighter color near **L4**/**L5** does not extend out as far; the contour lines are almost irrelevant in this) than the comparable region in **Figure 6a**, **p. 142**, with an "infinitesimal" 3rd body. Comparing **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**, suggests that a wide range of Trojan-ternary star systems or binary star systems with a planet at **L4** or **L5** are possible.

- ✓ NOTE: **L1** has moved to a place of symmetry between **L4** and **L5**, since the 2 major masses are equal. The distance from **L2** and **L3** to its nearest major mass is the same as the distance between the 2 major masses, symmetrical since the 2 major masses are equal.
- ✓ NOTE ALSO: a contour plot is like a topographical map: the shape of the contour lines depends on the "elevation" of the intersections of various "cutting planes" with the function's 3-dimensional surface. The contour lines of the same surface can look quite different if contour-plotted slightly differently. Some of the difference between the contours of **Figure 6a**, **p. 142**, and **Figure 6b**, **p. 144**, is merely that, some that the region of possible stability around the Trojan points is narrower when all 3 masses are equal.
- ✓ ALSO NOTE: technically, the Lagrangian points do not even exist in the case of 3 equal masses, since Lagrange's restriction, i.e. that $m < \sim 0.04\ M$, is not met.

To examine stability carefully, it would be necessary to look at the total dynamics of the system, but computers should make that quite feasible, even without making the simplifying assumption—currently considered necessary—of "infinitesimality" for any of the masses. Even if it is unstable, a Trojan-ternary star system could take an astronomically significant time to degrade. And it's good to remember: the scientists who think ternary star systems must be unstable and must therefore be impossible also think lighter and heavier bodies always fall at precisely the same rate.

(Plotted with MathCAD 2000.)

ABOUT THE AUTHOR

The author grew up with a love of science, mathematics and philosophy from a very early age. He got his BS degree in mathematics with a minor in physics (with honors) at the University of Illinois in 1967.

After graduate studies in computer science (U of I), he spent roughly 2 decades doing software development in various parts of the world, eventually succumbing to overwork in Silicon Valley and failing health, and retiring early.

During the prolonged, and still continuing, period of severe ill-health and inactivity that followed, the author decided to go back to his early loves of science and mathematics to see what he could come up with in the way of "science… when wrong, to be put right." He has found a number of important "… oversights", both in physics and in mathematics. *Newton's Great… Oversight* is among them.

ABOUT THE AUTHOR–FOR "STOUTLY GIFTED CHILDREN OF ANY AGE"

Family legend has it that, at the precise instant I was born, birth-certificated at 2:54AM, Central Standard Time, December 19, 1945, at Elmhurst Hospital, in Elmhurst, Illinois, the Christmas tree at my about to be new home, just south of the railroad tracks on Parkside, fell over. You might as well stop reading here because there is nothing more interesting than that in what follows.

I was a late bloomer, starting the talking and walking bits a few months *later* than just *average* average. I distinctly did *not* read Proust when I was 3! I don't remember reading anything—except probably comic books—until I was 5 years old. I started K when I was 4, but not because I was recognized as "gifted". It was really only the "one less noisy kid around the house during the day when a severely grouchy dad who worked at night was trying desperately to get some sleep" thing, that and my December 19th birthday that let me squeak in. I was never (visibly) a prodigy of any kind. (The "prod" in "prodigy" is an essential clue, n'est-ce pas?) I probably seemed at best a "bright underachiever", as one of my junior high teachers put it, genially trying to nudge me to do "better". I literally *never* studied very much *for school*, even in college, and I am not at all sure if I would have "done any better" if I *had* studied. And I was never "prodded" to do "prodigious" things, internally or externally, very few notable or even visible things anyway. "Brighter than average-average, but generally unprepossessing" was probably my usual semblance to others. Only little things… here, there…

I grew up with a love of science, mathematics and philosophy from a very early age. By age 7 I shyly asked my father if I could "skip ahead a few years in school", to which my father said no; reason: "worried about socialization". (I was by far the smallest in my age group, and already a year ahead in school.) I shyly repeated my request again at age 8; same response.

At this same age of 8 I quietly extended my self-study to dabbling in college level mathematics (this was in the early 1950s, and college level mathematics was *much* simpler then than it is today) and "cut my teeth" on Bertrand Russell's philosophical works, reading them cover to cover—and especially on Russell's stout English eccentricity and "non-establishment" orientation:

- "Have no respect for the authority of others, for there are always contrary authorities to be found."
- "Be scrupulously truthful, even if the truth is inconvenient, for it is more inconvenient when you try to conceal it."
- "Do not fear to be eccentric in opinion, for every opinion now accepted was once eccentric."
  Bertrand Russell
- ✓ Let me digress a bit to mention something concerning "authority" and a great danger to people who are "half wise". They often know that freedom is essential, so they rightly feel free to have no respect for the "all rule and all authority and power" of *others*, i.e. *authority etc that is not their own*. Notice the emphasis. It is their *own* "authority" etc that can become a deadly danger to them. They can wind up in a Chinese-style self-strangling "ligion" or "religation" of their own devising and handiwork that they can do nothing about… *because it is their own*. They foolishly feel they must "respect *it*"… *because it is their own*. Most often they wind up in an ugly death struggle against themselves, a guaranteed Pyrrhic victory–Pyrrhic loss situation.
  Not a good way to fly.

Russell distinctly helped launch my own deeply-rooted and equally stout English eccentricity that quietly lay in wait in my otherwise quite American DNA. (To a British reader, this would readily explain the eccentricity in my "formal credentials".)

My mother died in the spring when I was twelve: "… flowers came up anyway."

A year later, in the summer of 1959, what was left of my family (my two 9-and-8-years-older sisters had fledged and flown, so my father, my 1-year-younger sister, our dog, Corky, and I) migrated across the US in a 30 ft Airstream travel trailer-half-ton pickup truck combo, winding up nesting in Chandler, AZ, where I started high school. During my first months there, I all but invisibly demonstrated my "talent" on a slightly more formal basis, while taking (with no special prep whatsoever) a new experimental "every kind of bias we are afraid of free" IQ test given to my entire freshman class. The "Given this abstract pattern, which of these 4 other abstract patterns is most like it?" type questions were supposed to lack any significant language-culture-education-whatever bias. The results were only reported directly to the parents, not to teachers or students, because of the fear at that time of "tracking", which then meant that the teachers tended to spend the most time with the "best" students, a justified fear in Chandler at that time, as far as I can remember. My father, looking stricken and acting *very* strangely, reluctantly let me know I had—these days it is called—"ceilinged" the test, and made sure I kept this secret from everyone, for what may still be usual "psychological reasons". Because of my shyness and the distressing strangeness of my father's reaction, I never told my father that I had walked—almost dawdled, in *some* ways, spending a lot of time "daydreaming"—through the entire test in just a tad less than half the allotted time of 45 minutes.

Just in case this is interesting to some: because of my "talent" (like a calculator savant, but for patterns and their interrelations), it only took me on rough average maybe 3/4 of a second to a whole second to both look at the question *and* figure out *both* which answer I thought was correct *and* which answer I thought the question preparer thought was correct. Did I mention that my "daydreaming" consisted of consciously making, analyzing and resynthesizing (non-linguistic, quasi-instantaneous gestalt-gedanken) models of the cognitive processes of the *various* people who made up the questions, modeling their mostly *subconscious* models of the cognitive processes of the question answerers, making my own models of the cognitive processes of said question answerers (such as myself), etc?! One of the question preparers had a marked and to me disturbing pattern blindness. I disagreed maybe with 10 of his "answers", out of maybe 50 or so of his questions, and marked the answers "*my* way".

I was now secretly but officially a new, American species of "Traditional English Eccentric Genius". After that I knew for sure what I had merely hazily guessed at, that I fell in the "Un-Godly High IQ portion of the population", and somewhat ironically I have mostly found myself using it to try to understand other people, which is not nearly as easy as it sounds. I noticed *very* early in life that "I see things others don't", but I simultaneously realized that, even if they seem very slow to me, "other people see things I don't", that they live in whole other worlds that I can only guess at, and even my "talent" is not enough to guarantee anything in that regard. All this, combined with my Bertrand Russell-like English eccentricity, honestly come by through my DNA, and health that started failing-degenerating early on, has given me a different path through life…

I never skipped ahead in school like I had hoped. Sigh…

I finished 3 years of high school, 2 years in Chandler, AZ, and 1 year in Park Forest, IL. But I spent my senior year in Italy with the (then) American Field Service foreign exchange student program, for which I feel distinctly grateful every time I remember it! (Thank you, bless you, AFS!) I arrived back in the US in early August,

sans high school diploma, sans GED, and sans even the faintest sane plan for attending a "good university". (Did I mention I was more than a bit naïve?!) All I had were my disappointing SAT scores…

This brings up a flashback story. I took the SATs in Italy, in the spring of 1963, and under unusual circumstances. I was very sick a few weeks before the exams, and when I say very sick, I mean that I had a fever that went way over 106° F once, plus the usual before and after. I had been out of bed for a week, less than 2 weeks from the 106++° F when I took the (highly scheduled; few opportunities) exams. I occasionally have that kind of Irish (my mother was a Lunney) thick-headed willfulness. I did *not* want to miss those exams. I took 5 exams that day: the (then) usual two, the verbal and math aptitude tests, plus the physics, chemistry and advanced math achievement tests. The toll that illness took showed itself immediately: I only got one perfect score of 800, on the advanced math achievement test. The rest was a series of horrors: a 745 in chemistry; a truly horrifying 688 in physics (my intended major); the required math aptitude was another horror, 719, but it was the required verbal aptitude that told the story of what was *really* going on here: I got a 666¡ I'm sure I don't have to tell you what *that* means… I didn't know at the time how much of a clue that was to the future twists, turns and downward spirals of my life. As an example, the impairment of my facility with languages—including mathematics—turned out to be even more severe than that score suggests, and not only never went away, an average slow, punctuated degeneration process never went away either. It took me many years to get a realistic estimate of how badly that episode had damaged me, and continued to. End flash-back.

So, at the last minute—early August—my father took me to the Champaign-Urbana campus of the University of Illinois—distinctly a good university, even if not "Ivy League". Even though I met almost none of the usual formal requirements, I was admitted straight away into the James Scholar honors program, majoring in physics, at first. I was so naïve when I first got there that I didn't know about testing out of basic classes. When I found out, it was too late to skip analytic geometry, but I immediately tested out of 2 semesters of calculus, skipped advanced calculus, and went into an honors real analysis class.

My father died in the spring when I was twenty: "… Then, too, flowers came up anyway."

There, at the U of I, I got my BS in math (physics minor, just honors since I didn't study) in 3 academic years (6 semesters and 2 summers), in January of 1967 (having traditionally dropped out for a semester to "find myself"). But, alas, I fell victim to the All But Dissertations Curse while doing graduate work in computer science: my thesis advisor *totally* unexpectedly rejected my (brilliant, of course) dissertation topic, and simultaneously someone offered me $ to work as a consultant to design and program an operating system for a mini-computer; I "did the math". I moved on.

I spent roughly 2 decades doing software development in various parts of the world, including living and "consulting" in London, followed immediately by Silicon Valley (to try to get some *sun* back in my life), finally succumbing to a combo of Silicon Valley burnout and a devastatingly disabling case of CFIDS (officially diagnosed only over 10 years later) that had started in earnest 19 years earlier with mononucleosis in college, and retiring early, in my 39th year. That may have been symbolic, but it was not actually practical.

During the prolonged, and still continuing, period (now decades long) of death-defying ill-health and all-but complete inactivity that followed, I returned to my early loves of science, math and philosophy. I decided to use my "talent" to see what I could come up with that others had "… overlooked" in the way of "Science (and math)… when wrong, to be put right." In these last 2 decades I have found a number of

important such "… oversights", especially in set theory and physics. *Newton's Great… Oversight* is among them.

And you thought you were going to find out why the Christmas tree fell over…

Michael Hugh Knowles
Rua Miradouro, 45
Sion
30310-640, Belo Horizonte, MG, Brasil
mhk@mhknowles.net

[This page intentionally left blank.]

[BACK COVER]

A Novel Twist in the Modern Science Wars: Isaac Newton *and* Modern Science Failing on *Scientific* Grounds…

If we have 2 masses, 1 lighter, 1 heavier, $m_L < m_H$, and we release them at separate times from the top of the Tower of Pisa, the theoretical falling rates are the *combined* accelerations of each mass and the Earth toward each other, i.e.

the **lighter mass falling rate** = $G\frac{m_E}{r^2} + G\frac{m_L}{r^2}$ < $G\frac{m_E}{r^2} + G\frac{m_H}{r^2}$ = the **heavier mass falling rate**

and we can see that the lighter and heavier bodies have *different* falling rates.

But there's more…

Simple **Newtonian theory of gravity**
+
simple **algebra**
+
simple **trigonometry**
+
"*scientific heresy*"

(questioning Galileo's *scientific* hypothesis that lighter and heavier bodies fall at the *same* rate,
that for some strange reason we all still believe is scientifically correct)

=

a *simple* proof that Newton's theory of gravity *predicts*
that lighter and heavier bodies *must fall at different rates* (*usually*)
when released *simultaneously* from the top of the Tower of Pisa
+
a *simple* approach to the alluring astronomy of Lagrange's Trojan asteroids
and their *fascinating* "tadpole" and "horseshoe" orbits

without **using *exceptionally difficult* perturbation theory,**
without **using Lagrange's *exceptionally difficult* calculus of variations,**
without **using *extremely difficult* partial differential equations,**
and *without* using even *simple calculus!*

Newton *Himself* Made A Serious "… Oversight"

Popular Science at Its Most Fascinating!

IMPATIENT?!

WHERE YOU CAN FIND EQUATIONS:
Section 3.4 Some Basic Equations and Some Simple Equations starts on **p. 44**.
Section 3.5 Equations for a *Simple* 3-Body Problem starts on **p. 47**.
Section 3.9 Equations for Homographically Maintaining an Equilateral Triangle Without Expansion and Contraction starts on **p. 54**.

You, too, may find yourselves asking:

"*What* else *is Science missing?!*"